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FUZZY PRIME SUBMODULES AND RADICAL OF A FUZZY SUBMODULE

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Abstract. An attempt is made to fuzzify the concept of a prime submodule and that of the radical of a fuzzy submodule. A study of (i) the level submodules of a fuzzy prime submodule, (ii) the algebraic nature of fuzzy prime submodules under homomorphisms, and (iii) the radical of a fuzzy prime submodule, is carried out.

Throughout the paper, it is assumed that R is a commutative ring with unity. Further, M and M' denote modules over R.

1. Introduction. The purpose of this paper is to introduce and study the concept of a fuzzy prime submodule and the radical of a fuzzy submodule. It is shown that a fuzzy submodule θ of M is fuzzy prime iff each level submodule $\theta_t = \{x \in M | \theta(x) \geq t\}$, $t \in Im\ \theta$, is a prime submodule of M. Further, a one-to-one correspondence is established between the set of all f-invariant submodules of M and the set of all submodules of M'. The radical of a fuzzy submodule of M is shown to be a fuzzy ideal of the ring R. The radical of a fuzzy prime submodule is also examined and turns out to be a fuzzy prime ideal of the ring R.

2. Fuzzy Submodules and Fuzzy Prime Submodules

Pan (1987) has defined a fuzzy submodule as follows :

A fuzzy subset θ of M is called a *fuzzy submodule* of M if, for all $a, b, \in M$ and $r \in R$, the following conditions are satisfied :

(i) $\theta(a+b) \geq \min\{\theta(a), \theta(b)\}$,

(ii) $\theta(-a) = \theta(a)$,

(iii) $\theta(0) = 1$, and

(iv) $\theta(ra) \geq \theta(a)$

It is worth noting that codition (ii) follows from condition (iv) (since the ring R has unity). Also, in the results to follow, we shall not be requiring condition (iii). Thus, we have the following definition of a fuzzy submodule :

DEFINITION 2.1. *A fuzzy subset θ of M is called fuzzy submodule of M if, for all $a, b \in M$ and $r \in R$, the following conditions are satisfied :*

(i) $\theta(a+b) \geq \min\{\theta(a), \theta(b)\}$, *and*

(ii) $\theta(ra) \geq \theta(a)$

For a fuzzy submodule θ of M it can be easily verified that the lavel subset $\theta_t = \{a \in M | \theta(a) \geq t\}$, $t \in Im\theta$, are submodules of M.

DEFINITION 2.2. *Let θ be a fuzzy submodule of M and let $t \in [0, \theta(0)]$.Then the subodule θ_t is called a level submodule of θ.*

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