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ON FUZZY PRIMARY SUBMODULES

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Abstract : Fuzzy primary submodules are defined and studied in terms of their level submodules. Some properties of quotient of a fuzzy primary submodule and the intersection of two fuzzy primary submodules are discussed. Fuzzy cosets of fuzzy submodules are defined and the zero divisors in the module of all fuzzy cosets of a primary submodule are shown to be nilpotent.

Throughout the paper, R denotes a commutative ring with unity and M an R-module. Further it is assumed that the image set of every fuzzy submodule is finite.

1. Introduction. The concept of fuzzy submodules was first introduced by Negoita and Ralescu (1975) and subsequently studied, among others, by Fu Zheng Pan (1987).

In this paper, some properties of fuzzy primary submodules are studied. It turns out that a fuzzy submodule θ of M is fuzzy primary iff every level submodule of θ is primary except the whole module M (see Theorem 3.2). This result justifies the definition of the fuzzy primary submodule given in this paper. For a fuzzy submodule θ of M and an element $a \in M$, the fuzzy subset $(\theta : a)$ of R^{1}(see Definition 3.4) is shown to be a fuzzy ideal of R. In case, θ is fuzzy primary it turns out that $(\theta : a)$ is a fuzzy primary ideal of R (see Theorem 3.5). The radical of fuzzy ideal $(\theta : a)$ of R is also examined and under certain conditions, it is shown that $\sqrt{\theta : a} = \sqrt{\theta}$ [see Theorem 3.9]. A set of conditions under which the intersection of two fuzzy primary submodules is again a fuzzy primary submodule is also provided. Radical of intersection of two fuzzy primary submodules is also investigated.

Finally for a fuzzy primary submodule θ of M it is shown that every *zero divisor* in the module M_θ, consisting of all fuzzy cosets of θ in M, is nilpotent (see Theorem 4.4).

2. Preliminaries. In this section we give some basic definitions and results to be used later on.

DEFINITION 2.1. *Let μ be a fuzzy subset of a set S and let $t \in [0, 1]$. The set*

$$\mu_t = \{x \in S | \mu(x) \geq t\}$$

is called a level subset of μ.

DEFINITION 2.2. (Liu 1982). *A fuzzy subset μ of R is called a fuzzy ideal of R if*

(i) $\mu(x-y) \geq \min(\mu(x), \mu(y))$; *and*

(ii) $\mu(xy) \geq max(\mu(x), \mu(y))$ *for every* $x, y \in R$.

DEFINITION 2.3. (Kumar 1993) *A fuzzy ideal μ of R is called a fuzzy prime ideal of R if* $\mu(xy) \neq \mu(x)$ then $\mu(xy) = \mu(y), x, y \in R$.

DEFINITION 2.4. (Kumar 1993) *A fuzzy ideal μ of R is called fuzzy primary if for all x, y $\in$ R, either* $\mu(xy) = \mu(x)$ *or* $\mu(xy) \leq \mu(y^n)$ for some $n \in Z_+$.