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Trace Class Operators via OPV-Frames

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Abstract. Trace class operators for quaternionic Hilbert spaces (QHS) were studied by Moretti and Oppio [18]. In this paper, we study trace class operators via operator valued frames (OPV-frames). We introduce OPV-frames in a right quaternionic Hilbert space $\mathcal{H}$ with range in a two sided quaternionic Hilbert space $\mathcal{K}$ and obtain various results including several characterizations of OPV-frames. Also, we obtain a necessary and sufficient condition for a bounded operator on a right QHS to be a trace class operator which generalizes a similar result by Attal [2]. Moreover, we construct a trace class operator on a two sided QHS. Finally, we study quaternionic quantum channels as completely positive trace preserving maps and obtain various Choi-Kraus type representations of quaternionic quantum channels using OPV-frames in quaternionic Hilbert spaces.

1. Introduction

Duffin and Schaeffer [12] introduced frames in a study of non-harmonic Fourier series. Later, Daubechies, Grossmann and Meyer [11] reintroduced frames which gather a lot of attention among the researchers. The main reason for frames to be popular among researchers in recent years is their applications in digital signal processing [3] and other areas having physical and engineering problems [8].

Frames are integrally connected to time-frequency analysis. It is difficult to find a particular category of frames that is suitable to most of the physical problems, as there is no comprehensive class of frames that suits to all types of problems. Keeping this in mind, researchers across the disciplines come together for finding tools for the theory of frames to tackle various physical problems including solutions of operator equations in Hilbert spaces with fixed dual pairing [4]. Duffin and Schaeffer [12] defined frames as follows:

Let H be a Hilbert space. A sequence $\{x_n\}_{n\in\mathbb{N}} \subseteq H$ is a *frame* for H, if there exist numbers $A, B > 0$ such that

$$A\|x\|^2 \leq \sum_{n\in\mathbb{N}} |\langle x, x_n\rangle|^2 \leq B\|x\|^2, \quad x \in H. \tag{1}$$

The scalars A and B are called the *lower* and *upper frame bounds* of the frame, respectively. They are not unique. If $A = B$, then $\{x_n\}_{n\in\mathbb{N}}$ is called an *A-tight frame* and if $A = B = 1$, then $\{x_n\}_{n\in\mathbb{N}}$ is called a *Parseval*

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