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ON WEAVING $PG-$FRAMES

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Abstract Recently, a new application of frame theory in Hilbert spaces has been emerged related to distributed signal processing. Bemrose et al. [4] have developed a theory of weaving frames to handle such types of problems. In this article, we introduce and study woven $pg-$frames for Banach spaces. It has been shown that a family of $pg-$Bessel sequences is always woven Bessel family. The image of woven $pg-$frame under a bounded invertible operator is a woven $pg-$frame has been proved. Also, a necessary and sufficient condition for the image of woven $pg-$frame under a bounded operator to be a woven $pg-$frame has been given. Further, some characterizations of woven $pg-$ frames and characterization of woven $pg-$Bessel sequences are given. Furthermore, woven $qg-$Riesz bases are defined and prove that these are particular cases of woven $pg-$frames. Finally, some equivalent conditions for woven $qg-$Riesz basis are given.

1 Introduction

Duffin and Schaeffer [12] introduced the concept of frames in Hilbert spaces while studying the problems of non-harmonic Fourier series. They gave the following definition of frames in Hilbert spaces:

Definition 1.1. A family of vectors $\{x_n\}_{n\in\mathbb{N}}$ in a Hilbert space $\mathcal{H}$ is said to be a frame for $\mathcal{H}$, if there exist two constants $0 < A \leq B < \infty$ such that

$$A\|x\|^2 \leq \sum_{n\in\mathbb{N}} |\langle x, x_n\rangle|^2 \leq B\|x\|^2,\ \forall\, x \in \mathcal{H}.$$

Later, in 1986, Daubechies et al. [10] reintroduced frames and observed that frames can be used to approximate functions in $L^2(\mathbb{R})$. These days theory of frames become an integral and important tool to study the problems of applied mathematics and engineering. For nice introduction of frames, one may refer [8].

The concept of frames was extended to Banach spaces by Feichtinger and Gröchenig [13]. They introduced the notion of atomic decomposition for Banach spaces. Later, Gröchenig [14] introduced a more general concept for Banach spaces called Banach frames. He gave the following definition:

Definition 1.2. Let $\mathcal{X}$ be a Banach space and $\mathcal{X}_d$ an associated Banach space of scalar-valued sequences indexed by $\mathbb{N}$. Let $\{f_n\} \subset \mathcal{X}^*$ and $S : \mathcal{X}_d \to \mathcal{X}$ be given. Then, the pair $(\{f_n\}, S)$ is called a Banach frame for $\mathcal{X}$ with respect to $\mathcal{X}_d$, if

(i) $\{f_n(x)\} \in \mathcal{X}_d$, for each $x \in \mathcal{X}$.

(ii) there exist constants A and B with $0 < A \leq B < \infty$ such that

$$A\|x\|_{\mathcal{X}} \leq \|\{f_n(x)\}\|_{\mathcal{X}_d} \leq B\|x\|_{\mathcal{X}},\ \ x \in \mathcal{X}.$$

(iii) S is a bounded linear operator such that $S(\{f_n(x)\}) = x,\ \ x \in \mathcal{X}$.