ON FUSION FRAMES IN QUATERNIONIC HILBERT SPACES

S.K. Sharma, Nitin Sharma and Khole Timothy Poumai

Communicated by Harikrishnan Panackal

MSC 2010 Classifications: Primary 42C15; Secondary 42A38.

Keywords and phrases: Fusion frames, woven frames, quaternionic Hilbert space.

Abstract In [15] authors have studied frames of operators in Quaternionic Hilbert spaces and silently discussed that fusion frames are particular case of frames of operators. In this paper, we have studied and extended their properties and present some of their characterizations in quaternionic Hilbert spaces. Further, we have examined the existence of synthesis, analysis, and frame operators and investigated their properties for fusion frames in Quaternionic Hilbert spaces. Furthermore, we have given some perturbation results for fusion frames in Quaternionic Hilbert spaces similar to the results in Hilbert spaces. Finally, woven fusion frames in Quaternionic Hilbert spaces are studied.

1 Introduction

Frames [11] in Hilbert spaces were introduced in 1952 while studying the non-harmonic Fourier series. But their potential was realized by the researchers after the work done by Daubechies, Grossman, and Meyer [3], due to its vast applications in various fields like signal and image processing, sigma-delta quantization, filter bank theory, and wireless communication. For more details, one may refer to [6]. In recent years, many generalizations of frames were introduced and studied. One of a generalization which is much appreciated by the researchers is fusion frames in Hilbert spaces introduced by P. Casazza and G. Kutyniok [8]. Fusion frames are used in filter bank theory, time-frequency analysis, and signal and image processing. For further details, regarding the applications and properties of fusion frames and its extension see [8, 9, 1]. In [19] Bemrose et al. have defined and studied the properties of weaving frames in Hilbert spaces which are used in distributed signal processing.

Hamilton discovered the field of quaternion which is a generalization of complex numbers, it is a four-dimensional non-commutative real algebra. Quaternions are used to study rotation in the higher dimension, theory of relativity, Newtonian and quantum mechanics, and general relativity in which Lorentz transformation is given in terms of quaternions. For details regarding quaternions see [13]. In [10] R. Ghiloni et al. have extended the continuous functional calculus in the case of quaternionic Hilbert spaces.

Khokulan, Thirulogasanthar and Srisatkunarajah [4] have defined and studied frames in finite dimensional quaternionic Hilbert spaces. In [14], Sharma and Goel have studied frames in the separable right quaternionic Hilbert spaces. H. Ellouz [2] studied K-frames in right quaternionic Hilbert spaces and studied the invertibility of corresponding frames operators. Recently in [12] Ruchi et al. has defined OPV-frames in right quaternionic Hilbert spaces and woven frames, woven K-frames and K-fusion frames in right quaternionic Hilbert spaces were studied in [17, 5, 18].

In this paper, we have studied and extended the properties of fusion frames and present some of their characterizations in quaternionic Hilbert spaces. Further, we have examined the existence of synthesis, analysis, and frame operators and investigated their properties for fusion frames in quaternionic Hilbert spaces. Furthermore, we have given some perturbation results for fusion frames in quaternionic Hilbert spaces similar to the results in Hilbert spaces. Finally, woven fusion frames in quaternionic Hilbert spaces are studied.