Journal of Classical Analysis
Volume 24, Number 2 (2024), 133–152
doi:10.7153/jca-2024-24-07

DUALITY AND STABILITY OF OPERATOR VALUED FRAMES FOR QUATERNIONIC HILBERT SPACES

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Abstract. This paper aims to prove some significant properties and relations between operator valued frames (OV-frames) in quaternionic Hilbert spaces. Various properties concerning the dual of an OV-frame are proved and the precise form of the family of duals of an OV-frame is given. Moreover, we try to construct OV-frames with the help of some partial isometries and finally, the stability of OV-frames under some perturbation conditions is established in quaternionic Hilbert spaces.

1. Introduction and preliminaries

The notion of frames and their generalizations like fusion frames, g-frames etc. have been studied and developed rapidly in the past decade mainly due to its significant applications in signal processing and coding theory [1, 4, 5, 13, 14, 15]. Further, a generalization of the concept of vector-valued frame, that enables us to deal with the operators in a Hilbert space instead of its elements, was introduced by L. Găvruta [6]. The notion of OV-frames, provides a more general way of series expansion of elements that is very similar to frame decomposition and have immense applications in quantum computing, packets encoding and many more. For more details on OV-frames, readers can refer [7, 9, 10, 11].

OV-frames in quaternionic Hilbert spaces are defined in [2]. This paper aims to provide a few more results on OV-frames in a right-quaternionic Hilbert space and is structured as follows: With the aim of making this paper self-contained, we recall some basic definitions and results concerning OV-frames and quaternions in the remaining part of this section. In Section 2, we prove some properties and relations between OV-frames. Section 3 concerns mainly about the properties of the duals of an OV-frame. In Section 4, construction of some OV-frames with the help of some partial isometries is given and finally, the stability of OV-frames under some perturbation conditions is established in Section 5.

All through the paper, H denotes a right quaternionic Hilbert space and K a two sided quaternionic Hilbert space, $\mathbb{H}$ the set of all real quaternions, $\mathbb{N}$ the set of all natural numbers and $\mathscr{I}$ an index set. The set of all the bounded operators from H to K is denoted by $\mathscr{B}(\mathsf{H},\mathsf{K})$ and the identity operator on H is denoted by I_{H}. The set of all the quaternionic valued square summable sequences is given by the set $\ell^2_{\mathbb{H}} =$

Mathematics subject classification (2020): 42C15.
Keywords and phrases: Frames, operator valued frames, quaternions, perturbation.
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Paper JCA-24-07