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TOPOLOGICALLY STABLE AND PERSISTENT POINTS OF GROUP ACTIONS

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Abstract

In this paper, we introduce topologically stable points, persistent points, persistent property, persistent measures and almost persistent measures for first countable Hausdorff group actions of compact metric spaces. We prove that the set of all persistent points is measurable and it is closed if the action is equicontinuous. We also prove that the set of all persistent measures is a convex set and every almost persistent measure is a persistent measure. Finally, we prove that every equicontinuous pointwise topologically stable first countable Hausdorff group action of a compact metric space is persistent. In particular, every equicontinuous pointwise topologically stable flow is persistent.

1. Introduction

In [16], the author has studied topologically stable homeomorphisms of compact metric spaces. In [14], Lewowicz called a homeomorphism to be persistent if every orbit of the map can be seen through some actual orbit of every small enough perturbed system. Every topologically stable homeomorphism of a compact manifold is persistent but this need not be true when the phase space is a compact metric space [15, Example 1]. Recently, in [13] authors have introduced pointwise topologically stable homeomorphisms. First motivation of this paper comes from the relationship obtained between pointwise topological stability and persistent property of a homeomorphism in [8], [11]. Precisely, authors have proved that every equicontinuous pointwise topologically stable homeomorphism of a compact metric space is persistent.

In [5], authors have introduced topologically stable finitely generated group actions of compact metric spaces and in [6], authors have studied pointwise topologically stable finitely generated group actions. In [4], author has introduced GH-stable countable group actions which extends the notion of topologically GH-stable homeomorphisms [2] and GH-stable finitely generated group actions [7], [12] in more general setting where GH stands for Gromov-Hausdorff distance. Second motivation of this paper comes from the study of such stability for countable group actions and from the notion of persistent

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