Topology and its Applications 320 (2022) 108218

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

Stability theorems in pointwise dynamics

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ARTICLE INFO

Article history:
Received 26 July 2021
Received in revised form 8 June 2022
Accepted 9 August 2022
Available online 28 August 2022

MSC:
primary 37B25
secondary 37B65

Keywords:
Expansive
Shadowing
Stability
Borel measure

ABSTRACT

We introduce minimally expansive and GH-stable points for homeomorphisms on metric spaces and μ-uniformly expansive, μ-shadowable and strong μ-topologically stable points for Borel measures (with respect to a homeomorphism on a metric space). We prove that: (i) a minimally expansive shadowable point of a homeomorphism on a compact metric space is topologically stable and GH-stable; (ii) a μ-uniformly expansive μ-shadowable point for a Borel measure μ (with respect to a homeomorphism on a compact metric space) is strong μ-topologically stable.

1. Introduction

In the study of dynamical systems, the theory of shadowing property is a well established branch. A system with the shadowing property forces a numerically computed trajectory to follow an actual trajectory of the system. It plays a significant role in guaranteeing the positivity of the topological entropy [15, Theorem 3] and the topological stability [22, Theorem 4] of the system.

The notion of topological stability was first studied for Anosov diffeomorphisms by Walters in [21]. In [22], Walters has extended this notion for the class of homeomorphisms and has proved that expansive homeomorphisms with the shadowing property on compact metric spaces are topologically stable. An important component in the hypothesis of the Walters' stability theorem called as expansivity, was first studied by Utz in [20] for homeomorphisms on compact metric spaces. In [18], Reddy started studying expansive behavior of a map from local viewpoint and constructed a homeomorphism on a compact metric space which itself is not expansive but is expansive at each point. Another stronger variant of an expansive point has been introduced in [2] with the name of uniformly expansive point. In [2], the authors have proved that

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https://doi.org/10.1016/j.topol.2022.108218
0166-8641/