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SEQUENTIAL SHADOWING IMPLIES SPECTRAL DECOMPOSITION

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Abstract. We study chain recurrence for finitely generated group actions on metric spaces under the presence of the shadowing property. We introduce the sequential shadowing property for such actions and prove that this property implies the spectral decomposition property if the phase space is compact.

1. Introduction

The purpose of this paper is to study the spectral decomposition property of finitely generated group actions. This property was first proved by S. Smale [9] for Anosov diffeomorphisms and the result is popularly known as the spectral decomposition theorem. It states that the non-wandering set of Anosov diffeomorphisms can be decomposed into a union of finitely many disjoint closed invariant sets on each of which the diffeomorphism is transitive. The fact that Anosov diffeomorphisms are expansive [7, p. 108] and possess the shadowing property [3, p. 74] inspired Nobuo Aoki [1] to extend the spectral decomposition theorem to expansive homeomorphisms with the shadowing property on compact metric spaces. The shadowing property and the concept of expansivity are two of the most influential concepts in the development of topological dynamics. Most of

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