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Topologically Stable Equicontinuous Non-Autonomous Systems

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Abstract. We obtain sufficient conditions for commutative non-autonomous systems on certain metric spaces (not necessarily compact) to be topologically stable. In particular, we prove that: (i) Every mean equicontinuous, mean expansive system with strong average shadowing property is topologically stable. (ii) Every equicontinuous, recurrently expansive system with eventual shadowing property is topologically stable. (iii) Every equicontinuous, expansive system with shadowing property is topologically stable.

1. Introduction

In experiments, it is seldom possible to measure a physical quantity without any error. Therefore only those properties that are unchanged under small perturbations are physically relevant. In topological dynamics, a meaningful way to perturb a system is through a continuous map.

A homeomorphism (resp. continuous map) f on a metric space X is said to be topologically stable if for each $\epsilon > 0$, there exists a $\delta > 0$ such that if h is another homeomorphism (resp. continuous map) on X satisfying $d(f(x), h(x)) < \delta$, for each $x \in X$, then there exists a continuous map $k : X \to X$ satisfying $f \circ k = k \circ h$ and $d(k(x), x) < \epsilon$, for each $x \in X$.

This particular concept of stability is popularly known as topological stability which was originally introduced for a diffeomorphism on compact smooth manifold [15]. By looking at the significance of the concept, it is worth to identify those dynamical properties which imply topological stability. One of such result in topological dynamics is "Walters stability theorem" which states that expansive homeomorphisms with shadowing property on compact metric spaces are topologically stable [16].

The notion of expansivity expresses the worse case unpredictability of a system. Although such unpredictable behaviour of symbolic flows was recognized earlier, the concept of expansivity for homeomorphisms on general metric spaces was introduced in the middle of the twentieth century [14]. The expansive behaviour of continuous maps is popularly known as positive expansivity [4].

For a continuous map f on a metric space X and fixed $x_0 \in X$, identifying those $x \in X$ whose orbit follow that of x_0 for a long time and hence understanding the asymptotic behaviour of $f^n(x)$ relative to $f^n(x_0)$ can provide deep insight of the system. Anosov closing lemma provides us with such information for a differentiable map on compact smooth manifold [1]. The notion of shadowing property originated

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