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Persistence and expansivity through pointwise dynamics

Abdul Gaffar Khan and Tarun Das

Department of Mathematics, Faculty of Mathematical Sciences, University of Delhi, Delhi, India

ABSTRACT
Using the notion of topologically stable points, it is proved that every equicontinuous pointwise topologically stable homeomorphism of a compact metric space is persistent. Also, using the notion of strong topologically stable points of a Borel probability measure, it is shown that every pointwise strong topologically stable Borel probability measure with respect to an equicontinuous homeomorphism of a compact metric space is strong persistent. Further, it is established that any homeomorphism of [0, 1] as well as that of (0,1) does not admit any uniformly expansive point. Finally, these results are used to show that the unit circle does not admit any expansive homeomorphism.

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1. Introduction

In [16], Walters has studied the notion of topologically stable homeomorphisms of compact metric spaces. In [9], authors have introduced pointwise topologically stable homeomorphisms of compact metric spaces as a local concept of the topological stability. They have proved that every topologically stable homeomorphism of a compact metric space is pointwise topologically stable and the converse is true for expansive homeomorphisms of compact manifolds [15]. The notion of topological stability is closely related to the notion of persistence also [10]. In fact, these notions are equivalent in the class of group of automorphisms of solenoidal group. Also, every topologically stable homeomorphism of a compact manifold is persistent but this implication need not be true for homeomorphisms of compact metric spaces [14]. In [6, Theorem 1], authors have proved that this implication holds true for equicontinuous homeomorphisms. Precisely, they have proved the following:

Theorem 1.1: *Let $f : X \to X$ be an equicontinuous homeomorphism of a compact metric space X. If f is pointwise topologically stable, then f is persistent.*

In [6], authors have decomposed the persistence of a homeomorphism into the corresponding properties for Borel probability measures. Precisely, they have studied the persistence using the notions of almost persistent measures and strong persistent measures

CONTACT Tarun Das tarukd@gmail.com