Bull Braz Math Soc, New Series (2020) 51:1001–1016
https://doi.org/10.1007/s00574-019-00178-5

Pointwise Dynamics Under Orbital Convergence

Abdul Gaffar Khan[1] · Pramod Kumar Das[2] · Tarun Das[1]

Received: 16 June 2019 / Accepted: 4 November 2019 / Published online: 20 November 2019

Abstract
We obtain sufficient conditions under which the limit of a sequence of functions exhibits a particular dynamical behaviour at a point like expansivity, shadowing, mixing, sensitivity and transitivity. We provide examples to show that the set of all expansive, positively expansive and sensitive points are neither open nor closed in general. We also observe that the set of all transitive and mixing points are closed but not open in general. We give examples to show that properties like expansivity, sensitivity, shadowing, transitivity and mixing at a point need not be preserved under uniform convergence and properties like topological stability and α-persistence at a point need not be preserved under pointwise convergence.

Keywords Expansivity · Shadowing · Transitivity · Topological Stability · Chaos

Mathematics Subject Classification Primary 54H20 ; Secondary 40A30

1 Introduction

The idea of studying the behaviour of a dynamical system from pointwise viewpoint was initiated by Reddy. In the process of answering a question posed by Gottschalk to him, he introduced and studied pointwise expansivity, a strictly weaker notion than expansivity (Reddy 1970). The power and the beauty of pointwise dynamics got highlighted in the recent works including (Akin 1996; Morales 2016; Ye and Zhang

✉ Tarun Das
tarukd@gmail.com

Abdul Gaffar Khan
gaffarkhan18@gmail.com

Pramod Kumar Das
pramodkumar.das@nmims.edu

[1] Department of Mathematics, Faculty of Mathematical Sciences, University of Delhi, Delhi, India

[2] School of Mathematical Sciences, Narsee Monjee Institute of Management Studies, Vile Parle, Mumbai 400056, India