Acta Polytechnica **62**(1):80–84, 2022
https://doi.org/10.14311/AP.2022.62.0080

ON THE WESS-ZUMINO MODEL: A SUPERSYMMETRIC FIELD THEORY

Daya Shankar Kulshreshtha[a,*], Usha Kulshreshtha[b]

[a] *University of Delhi, Department of Physics and Astrophysics, Delhi – 110007, India*
[b] *University of Delhi, Kirori Mal College, Department of Physics, Delhi – 110007, India*
[*] corresponding author: dskulsh@gmail.com

Abstract. We consider the free massless Wess-Zumino Model in 4D which describes a supersymmetric field theory that is invariant under the rigid or global supersymmetry transformations where the transformation parameter ϵ (or $\bar{\epsilon}$) is a constant Grassmann spinor. We quantize the theory using the Hamiltonian and path integral formulations.

Keywords: Wess-Zumino Model, supersymmetric field theories, Hamiltonian and path integral quantization.

1. Introduction

Supersymmetry (SUSY) is a symmetry that rotates bosons into fermions and fermions into bosons. It is one of the beautiful symmetries of nature. Also, a field theory (FT) which remains invariant under the rigid or global supersymmetry transformations (where the transformation paremeter is a constant Grassman spinor) and which also satisfies the super Poincare algebra (SPA) is usually referred to as a supersymmetric field theory (SFT). In this article, we consider the free massless Wess-Zumino Model (WZM) in 4D which describes a SFT. It may be important to mention here that the WZM is the first known example of an interacting 4D quantum field theory with linearly realised SUSY, studied by Wess and Zumino using the dynamics of a single chiral superfield (composed of a complex scalar and a spinor fermion). It may be important to mention that the WZM represents a typical SFT which is of central importance in the theory of SUSY, supergravity and superstring theory (SST) and for further details we refere to the work of Refs. [1–8].

The WZM describes an example of a non-manifest supersymmetry [5]. One could of course go to the formalism of superspace and superfields to construct a theory that has a manifest supersymmetry [5]. Taking this theory as an example, it is possible to formulate supersymmertic field theories in different dimensions including in higher dimensions. The WZM also provides a basic framework for the study of Ramond Nievue Schwarz (RNS) SST [8] which is an example of a SST with non-manifest SUSY. Further, starting with the WZM, it is also possible to construct a supergravity theory [1–6, 8].

SPA is a graded Lie algebra that includes anticommutation relations (ACR's) involving the supercharge Q_a – the generator of the SUSY transformations. WZM is one of the simplest examples of a SFT. In this article, we discuss the supersymmetry of WZM and present some remarks with respect to the rigid or global supersymmetry versus the local supersymmetry (which happens to be a Supergravity theory). Finally we consider the constraint quantization of this theory [7]. It is important to mention that the supersymmetry has profound applications in conformal hadron physics from light-front holography where it even has some observational prospects [9–11].

As mentioned above, the supersymmetry is a symmetry that relates bosonic and fermionic variables (or the bosons and fermions) so that:

$$\delta B = \bar{\epsilon} F \ , \quad \delta F = \epsilon \, \partial B \ ; \quad \partial \equiv \partial_\mu \tag{1}$$

Here, δ is bosonic, B is bosonic and F is fermionic. The transformation parameter ϵ (or $\bar{\epsilon}$) is a constant Grassman spinor and is fermionic. Grassman variables are anti-commuting. Supergravity theory on the other hand is a theory that has "local supersymmetry" and it is invariant under local Susy transformations where the transformation parameter depends on the spacetime x^μ. So the transformation parameter for supergravity: $\epsilon(x^\mu)$ or $\bar{\epsilon}(x^\mu)$ depends on x^μ and hence supergravity is a "gauge theory" of gravity. In contrast to this the WZM is a supersymmetric FT with rigid or global (not local) Supersymmetry.

Let us us consider two consecutive infinitesimal rigid supersymmetry transformations of a bosonic field B:

$$\delta_1 \ B = \bar{\epsilon}_1 F \ , \quad \delta_2 F = \epsilon_2 \partial B \tag{2}$$

This then implies that the two internal SUSY transformations lead us to a spacetime translation:

$$\{\delta_1, \delta_2\} B = a^\mu \partial_\mu B \ ; \quad a^\mu = (\bar{\epsilon}_2 \gamma^\mu \epsilon_1) \tag{3}$$

Presence of a spacetime derivative of B on right hand side (RHS) of above equation suggests that the Susy is an extension of the Poincare spacetime symmetry:

$$\{Q_a, \bar{Q_b}\} = 2(\gamma^\mu)_{ab} P_\mu \tag{4}$$