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MAXWELL-CHERN-SIMONS-HIGGS THEORY

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Abstract.
We consider the three dimensional electrodynamics described by a complex scalar field coupled with the U(1) gauge field in the presence of a Maxwell term, a Chern-Simons term and the Higgs potential. The Chern-Simons term provides a velocity dependent gauge potential and the presence of the Maxwell term makes the U(1) gauge field dynamical. We study the Hamiltonian formulation of this Maxwell-Chern-Simons-Higgs theory under the appropriate gauge fixing conditions.

Keywords: Electrodynamics, Higgs theories, Chern-Simons-Higgs theories, Hamiltonian formulations, gauge-theories.

1. Introduction

We study the Hamiltonian formulation [1] of the three dimensional (3D) electrodynamics [2–22], involving a Maxwell term [20], a Chern-Simons (CS) term [19, 21, 22], and a term that describes a coupling of the U(1) gauge field with a complex scalar field in the presence of a Higgs potential [22]. Such theories in two-space one-time dimension ((2+1)D) can describe particles that satisfy fractional statistics and are referred to as the reletivistic field theoretic models of anyons and of the anyonic superconductivity [21, 22].

A remarkable property of the CS action [21, 22], is that it depends only on the antisymmetric tensor $\epsilon^{\mu\nu\lambda}$ and not on the metric tensor $g^{\mu\nu}$. As a result, the CS action in the flat spacetime and in the curved spacetime remains the same [21, 22]. Hence CS action, in both the Abelian and in the non-Abelian cases represents an example of a topological field theory [21, 22].

The systems in two-space, one-time dimensions (2+1)D (i.e., the planar systems, display a variety of peculiar quantum mechanical phenomena ranging from the massive gauge fields to soluble gravity [19–22]. These are linked to the peculiar structure of the rotation group and the Lorentz and Poincare groups in (2+1)D. The 3D electrodynamics models with a Higgs potential, namely, the Abelian Higgs models involving the vector guage field A^μ with and without the topological CS term in (2+1)D have been of a wide interest [19–22].

When these models are considered without a CS term but only with a Maxwell term accounting for the kinetic energy of the vector gauge field and they represent field-theoretical models which could be considered as effective theories of the Ginsburg-Landau-type [22] for superconductivity. These models in (2+1)D or in (3+1)D are known as the Nielsen-Olesen (vortex) models (NOM) [20]. These models are the relativistic generalizations of the well-known Ginsburg-Landau phenomenological field theory models of superconductivity [2, 20, 22].

The effective theories with excitations, with fractional statistics are supposed to be described by gauge theories with CS terms in (2+1)D and a study of these gauge field theories and the models of quantum electrodynamics involving the CS term represent a broad important area of investigation [21, 22].

The CS term provides a velocity dependent gauge potential [21, 22], and the presence of the Maxwell term in the action makes the gauge field dynamical [20]. We study the Hamiltonian formulation [1] of this Maxwell-Chern-Simons-Higgs theory under the appropriate gauge fixing conditions [20, 22].

The quantization of field theory models with constraints has always been a challenging problem [1]. Infact, any complete physical theory is a quantum theory and the only way of defining a quantum theory is to start with a classical theory and then to quantize it [1]. Theory presently under consideration is also a constrained system. In the present work, we quantize this theory using the Dirac's Hamiltonian formulation [1] in the usual instant-form (IF) of dynamics (on the hyperplanes defined by: $x^0 = t =$ constant) under appropriate gauge-fixing conditions (GFC's) [1, 19–22].

2. Hamiltonian formulation

The Maxwell Chern-Simons Higgs Theory in two space one time is defined by the following action:

$$S = \int \mathcal{L}(\Phi, \Phi^*, A^\mu)\ d^3x, \tag{1}$$