Charged Compact Boson Stars and Shells in the Presence of a Cosmological Constant

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In this work we study the boson stars and boson shells in a theory involving *massive* complex scalar fields coupled to the U(1) gauge field and gravity in a conical potential in the *presence* of a cosmological constant Λ which we treat as a free parameter taking positive and negative values and thereby allowing us to study the theory in the de Sitter and Anti de Sitter spaces respectively. Boson stars are found to come in two types, having either ball-like or shell-like charge density. We have studied the properties of these solutions and have also determined their domains of existence for some specific values of the parameters of the theory. Similar solutions have also been obtained by Kleihaus, Kunz, Laemmerzahl and List in a theory involving *massless* complex scalar fields coupled to the U(1) gauge field and gravity in a conical potential in the *absence* of a cosmological constant Λ.

I. INTRODUCTION

Boson stars and boson shells representing the localized self-gravitating solutions were introduced long ago [1–3] and they have been studied vary widely in the literature [4–28]. Such theories are being considered in the presence of positive [14–17] as well as negative [15, 17–21] values of the cosmological constant Λ. The theories with positive values of Λ (corresponding to the de Sitter (dS) space) are relevant from observational point of view as they describe a more realistic description of the compact stars in the universe since all the observations seem to indicate the existence of a positive cosmological constant. Such theories are also being used to model the dark energy of the universe. However, the theories with negative values of Λ (corresponding to the Anti de Sitter (AdS) space) are meaningful in the context of AdS/CFT correspondence [29–31].

In Ref. [16], we studied the boson stars and boson shells in a theory of complex scalar field coupled to $U(1)$ gauge field A_μ and the gravity in the presence of a *positive* cosmological constant Λ (i.e. in the dS space) and in Ref. [15], we studied the *boson stars* in a theory of complex scalar field coupled to $U(1)$ gauge field A_μ and the gravity in the presence of a positive as well as negative cosmological constant Λ allowing us to study the theory in the dS as well as in the AdS space.

In the present work we study *not only the boson stars but also the boson shells* in this theory of complex scalar field coupled to $U(1)$ gauge field A_μ and the gravity and a cosmological constant Λ which we treat as a free parameter and *which takes positive as well as negative values* and thereby allowing us to study the theory in the dS as well as in the AdS space. As in Ref. [15], for our present investigations also we study the theory in the presence of a potential: $V(|\Phi|) := (m^2|\Phi|^2 + \lambda|\Phi|)$ (with m and λ being constant parameters). We investigate the properties of the solutions of this theory and determine their domains of existence for some specific values of the parameters of the theory.

Similar solutions have also been obtained by Kleihaus, Kunz, Laemmerzahl and List in a theory involving *massless* complex scalar fields coupled to the U(1) gauge field and gravity in a conical potential in the *absence* of a cosmological constant Λ [10, 11]. They have obtained explicitly the domain of existence of compact boson stars and boson shells. They have also considered the boson shells, which do not have an empty inner region $r < r_i$, but instead they harbour a Schwarzschild black hole or a Reissner-Nordström black hole in the region $r < r_i$ [10, 11]. Boson stars have also been studied in the presence of polynomial potentials [22–25]

In the present work, we construct the boson star and boson shell solutions of this theory numerically and we study their properties, where we assume the interior of the shells to be empty space (dS-like or AdS-like). The action and the equations of motion are given in section II. In section III the equations of motion are re-expressed in terms of the rescaled variables. The boundary conditions and the global charges are considered in section IV. The numerical solutions for boson stars and boson shells are studied in section V and finally the summary and conclusions are given in section VI.

II. THE ACTION, ANSÄTZE AND EQUATIONS OF MOTION

The action of the theory under consideration reads:

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\begin{aligned}
S &= \int \left[\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_M\right]\sqrt{-g}\; d^4x\,, \\
\mathcal{L}_M &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - (D_\mu\Phi)^*\,(D^\mu\Phi) - V(|\Phi|)\,, \\
D_\mu\Phi &= (\partial_\mu\Phi + ieA_\mu\Phi)\,, \quad F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)\,. \qquad (1)
\end{aligned}
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