PRAMANA
— journal of
physics

Vol. 74, No. 1
January 2010
pp. 27–37

Effect of curvature on a statistical model of quark-gluon-plasma fireball in the hadronic medium

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MS received 11 February 2009; revised 3 June 2009; accepted 30 July 2009

Abstract. The free energy of a quark-gluon plasma fireball in the hadronic medium is calculated in the Ramanathan *et al* statistical model after incorporating the effect of curvature. The result with the inclusion of curvature is found to produce significant improvements in all the parameters we calculated with respect to the earlier results. The surface tension with this curvature effect is found to be $0.17T_c^3$, which is two times the earlier value of surface tension which is $0.078T_c^3$, and this new result is nearly close to the lattice value $0.24T_c^3$. As far as transition is concerned, a thermodynamic variable like entropy shows weakly first-order phase transition and it shows continuity in the behaviour of specific heat.

Keywords. Quark-gluon plasma; quark-hadron phase transition.

PACS Nos 12.38.Mh; 21.65.+f; 25.75.Ld

1. Introduction

Quantum chromodynamics predicts [1] a phase transition from a deconfined to a confined matter of hadrons during the early stages of the Universe. In the beginning of the Universe it was considered to be in the form of deconfined matter of quarks and gluons, and subsequent process of cooling leads to transformation into hadrons. It is indeed a very complicated phenomenon as predicted by the heavy-ion collider experiments. So, the formation of quark-gluon plasma (QGP) droplet (fireball) in ultrarelativistic heavy-ion collisions is a very exciting field in heavy ion collider physics [2]. The approach of central assumption treats the QGP-hadron system as a quasistatic equilibrium enabling applicability of equilibrium statistical mechanics to this complicated system. This central assumption has been employed by many pioneers in this field [2,3].

As we know from the extensive literature [3], we really need to look at the nucleation process produced by the statistical fluctuations of the critical free energy difference between two phases. The nucleation was explained by the model of Csernai–Kapusta *et al* [3–5] and they used it as liquid drop model expansion for this nucleation. This model was used and modified by Ramanathan *et al* [4] to calculate the free energy of QGP droplet formation, and subsequently produced surface tension and weakly transition at $T = (165 \pm 5)$ MeV. With these arguments, it is again modified with the inclusion of curvature term in their free energy expansion of the liquid drop. The modified free energy is

$$\Delta F = -\frac{4\pi}{3}R^3[P_{\rm had}(T,\mu_{\rm B}) - P_{\rm q,g}(T,\mu_{\rm B})] + 4\pi R^2\sigma + 8\pi CR. \tag{1}$$

The first term represents the volume contribution, the second term is the surface contribution where σ is the surface tension, and the third term is the curvature term.

But in Ramanathan *et al*'s approximation, the total free energy is obtained through the relativistic density of states for quarks and gluons. In this paper, after incorporating the curvature effect, it again becomes imperative to adapt the density of states for quarks and gluons. The adaptation in the density of states is done in such a way that it can give better results in the calculation while calculating the free energy contributed by quarks and gluons. It is being facilitated with the consideration of higher-order approximation as suggested by many authors [6,7] to give higher potential barrier. To implement the overall idea, we construct density of states briefly highlighting the Ramanathan *et al* statistical model in the hadronic medium. In further sections, we use this constructed total free energy to calculate the interfacial surface tension, thermodynamic variables like entropy, specific heat and lastly we give the comparative results with other models [14,17].

2. Detemination of density of states for the QGP droplet with curvature term

To determine the density of states of a QGP droplet, the atomic model of large atomic number of Thomas and Fermi [8] is modified by Ramanathan *et al* [4]. The total electronic density of states defined by Thomas and Fermi in phase space is

$$\int \rho_{\rm e}(k){\rm d}k = [-2mV(k)]^{3/2}\nu/3\pi^2 \tag{2}$$

or

$$\rho_{\rm e}(k) = [\nu(2m)^{3/2}/2\pi^2][-V(k)]^{1/2} \cdot \left[-\frac{{\rm d}V(k)}{{\rm d}k}\right]. \tag{3}$$

This model of electron is replaced by Ramanathan *et al*'s model of QGP droplet with the corresponding density of states for quark and gluon with a suitable QCD-induced phenomenological potential $V(k)$. It is given as

$$\int \rho_{\rm q,g}{\rm d}k = [-V_{\rm conf}(k)]^3\nu/3\pi^2 \tag{4}$$

or

$$\rho_{\mathrm{q,g}}(k) = (\nu/\pi^2)\left\{(-V_{\mathrm{conf}}(k))^2\left(-\frac{\mathrm{d}V_{\mathrm{conf}}(k)}{\mathrm{d}k}\right)\right\}_{\mathrm{q,g}}, \tag{5}$$

where ν is the volume occupied by the QGP and k is the relativistic four-momentum in natural units. $V_{\mathrm{conf}}(k)$ could be any confining potential for quarks and gluons. This potential plays the role of a mean-field potential in phase space similar to the mean-field potential of the Thomas–Fermi scheme, but in a very different context, namely, the QGP–hadron phase transition. Using these two models, the density of states is modified with a suitable parametrization factor in the dynamics of the QGP fluid. As suggested earlier that the curvature is an essential higher-order correction factor in free energy expansion, we use the technique of Neergaard *et al* [7] to modify the density of states so that it gives more effective results in the free energy. So, the modified density of states for quarks and gluons is given as

$$\rho(k) = \rho_{\mathrm{q,g}}(k) + \rho(k)_{\mathrm{cur}},$$

where

$$\rho(k)_{\mathrm{cur}} = C_i \int \mathrm{d}s \left(\frac{1}{R_1} + \frac{1}{R_2}\right). \tag{6}$$

Here, C_i is a function of k/m which depends on the type of field and the boundary conditions. The different values of C_i for different particles are given as

$$C_s = \frac{1}{12\pi^2}\left[1 - \frac{3k}{2m}\left(\frac{\pi}{2} - \arctan\left(\frac{k}{m}\right)\right)\right], \tag{7}$$

$$C_g = -\frac{1}{6\pi^2} \tag{8}$$

and

$$C_u = C_d = -\frac{1}{24\pi^2}, \tag{9}$$

which are in the limit of dynamical quark mass.

3. The free energy through mean-field interquark potential

To calculate the interacting potential within the system, it is very ideal to find the effective mean-field potential among the quarks and quark-gluon. This effective potential is called phenomenological potential and it is obtained through thermal Hamiltonian for the QGP [9] and is given as

$$V_{\mathrm{eff}}(k) = (1/2k)\gamma_{\mathrm{g,q}} g^2(k) T^2 - \frac{m_0^2}{2k}, \tag{10}$$

where $g(k)$ is the first-order QCD running coupling constant, which for quarks with three flavours is

$$g^2(k) = (4/3)(12\pi/27)\{1/\ln(1 + k^2/\Lambda^2)\}. \tag{11}$$

In the above expression, $\Lambda = 150$ MeV is the QCD parameter with our parametrization factors $\gamma_{\mathrm{g,q}}$ with $\gamma_{\mathrm{q}} = 1/6$ and $\gamma_{\mathrm{g}} = 8$ or 6 times γ_{q}. This value fits the lattice QCD simulations [10]. This effective perturbed potential will have a minimum value at each point of phase space. That is

$$V(k_{\min}) = (\gamma_{\mathrm{g,q}} N^{1/3} T^2 \Lambda^2/2)^{1/4}, \tag{12}$$

where $N = (4/3)(12\pi/27)$.

So, this value is called the low energy cut-off in the model leading to finite integrals by avoiding the infrared divergence. It is of the same order of magnitude as Λ and T. Now we get the modified free energy for quarks and gluons with the above modified density of states obtained in the above section through the effective potential and it is

$$F_i = \mp T g_i \int \mathrm{d}k \rho_i(k) \ln(1 \pm \mathrm{e}^{-(\sqrt{m_i^2+k^2})/T}), \tag{13}$$

where $\rho_i(k)$ is the modified density of states of the particle i (quarks, gluons, pions etc.) and g_i is the appropriate degeneracy factor (colour and particle–antiparticle degeneracy) which is 6 for quarks, 8 for gluons [7] and 3 for pions. The interfacial energy obtained through a scalar Weyl surface in Ramanathan *et al* [4,6,11] with suitable modification to take care of the hydrodynamic effects [12] is given as

$$F_{\mathrm{interface}} = \frac{1}{4}\gamma R^2 T^3, \tag{14}$$

where

$$\gamma = \sqrt{2} \times \sqrt{(1/\gamma_{\mathrm{g}})^2 + (1/\gamma_{\mathrm{q}})^2}, \tag{15}$$

which is the effective rms value of the flow parameter of the quarks and gluons respectively. The pion free energy is [6]

$$F_\pi = (3T/2\pi^2)\nu \int_0^\infty k^2 \mathrm{d}k \ln(1 - \mathrm{e}^{-\sqrt{m_\pi^2+k^2}/T}). \tag{16}$$

For the quark masses we use the current (dynamic) quark masses $m_0 = m_d = 0$ MeV and $m_s = 150$ MeV just as in ref. [6].

We can thus compute the total modified free energy F_{total} as

$$F_{\mathrm{total}} = \sum_i F_i + F_{\mathrm{interface}} + F_\pi, \tag{17}$$

where i stands for u, d and s quarks and gluons.

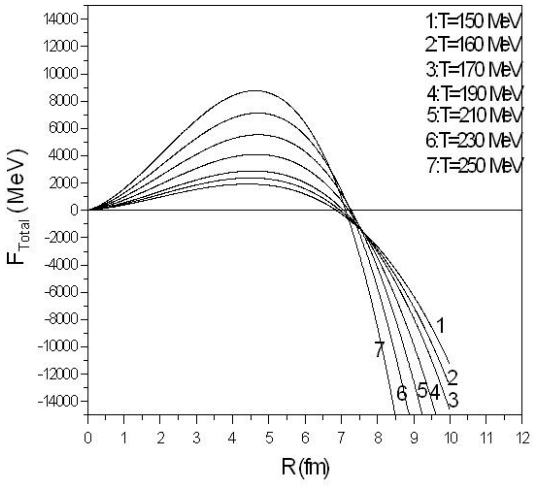

Figure 1. Modified $F_{\rm total}$ at $\gamma_{\rm g} = 6\gamma_{\rm q}$, $\gamma_{\rm q} = 1/6$ for various temperatures.

4. Modified surface tension

To calculate the surface tension of the fireball, we use the relation $\Delta F_{\rm c} = F(R_{\rm c}, T) - F(0, T)$ in eq. (1). From this, we obtain the surface tension with the effect of curvature as

$$\sigma = \frac{2}{R_{\rm c}} \left(\frac{3\Delta F_{\rm c}}{8\pi R_{\rm c}} - 2C \right), \tag{18}$$

where C is the curvature coefficient and $R_{\rm c}$ is the critical radius of the fireball. This critical radius $R_{\rm c}$ can be obtained by minimizing eq. (1) with respect to the droplet radius R, which gives two critical radii,

$$R_{\rm c} = \frac{\sigma}{\Delta p} \left(1 \pm \sqrt{\left(1 + \frac{2\Delta p C}{\sigma^2} \right)} \right) \tag{19}$$

with smaller radius corresponding to a local minimum in free energy and the larger to a local maximum. We take the local maximum value of the radius as suitable for our fireball system. At the local maxima, it has existence of stable solution. So, the free-energy change with respect to the droplet radius is calculated through our modified free energy given in eq. (17). Now, the calculation of surface tension is performed for the two parametrization factors $\gamma_{\rm g} = 6\gamma_{\rm q}$ and $\gamma_{\rm g} = 8\gamma_{\rm q}$. These two factors give more exciting results in the free energy which are shown in figures 1 and 2. In these two figures, the formation of droplet with observable critical droplet radius exhibits proper transition features in the band of temperature 150 MeV to 170 MeV which is expected from lattice calculations. Again, these two figures show measurable droplet radius of the order of few Fermi, and indicate significant barrier amplitudes in free energy which account for the nucleation rate of droplet formation. So, in the calculation of the surface tension, the values of the critical

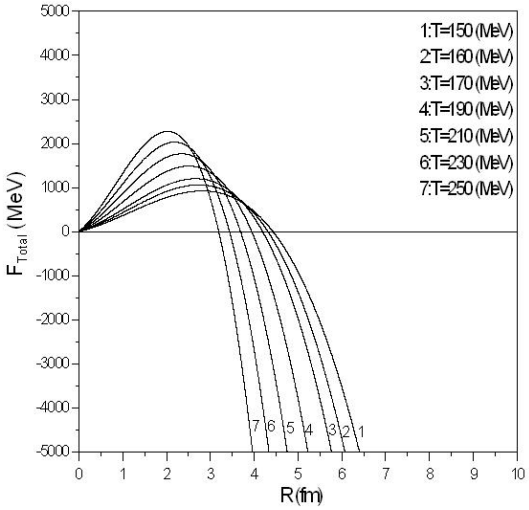

Figure 2. Modified $F_{\rm total}$ at $\gamma_{\rm g} = 8\gamma_{\rm q}$, $\gamma_{\rm q} = 1/6$ for various temperatures.

Table 1. Modified surface tension of QGP droplet at $\gamma_{\rm g} = 8\gamma_{\rm q}$, $\gamma_{\rm q} = 1/6$.

$T_{\rm c}$ (MeV)	$\Delta F_{\rm c}$ (MeV)	$R_{\rm c}$ (fm)	σ (MeV/fm^2)	$\sigma/T_{\rm c}^3$
150	921.93	2.814	14.798	0.17
160	1059.00	2.756	17.949	0.17
170	1200.00	2.665	21.528	0.17
190	1486.00	2.506	30.083	0.17
210	1765.00	2.344	40.586	0.17
230	2030.00	2.182	53.365	0.17
250	2272.00	2.024	68.540	0.17

free energy at the corresponding critical fireball radius are extracted from these two figures. Then, we compute the modified surface tension of the fireball using eq. (18), which are listed in tables 1 and 2. The constant values of $\sigma/T_{\rm c}^3$ show that the surface tension is independent of transition temperature and values of the parametrization factors. It is two times the earlier value of surface tension.

5. Thermodynamic variables and the phase transition

The thermodynamic properties like entropy and specific heat can be related from the Helmholtz free energy obtained in eq. (17). These entities are calculated for these two parametrization factors $\gamma_{\rm g} = 6\gamma_{\rm q}$ and $\gamma_{\rm g} = 8\gamma_{\rm q}$. The behaviour of entropy, S vs. temperature, T indicates the real nature of the phase transition of the system. These output characteristics shown in figures 3 and 4 indicate that there exists a very weak discontinuity in the vicinity of $T_{\rm c} = 160$ MeV in the entropy

Table 2. Modified surface tension of QGP droplet at $\gamma_g = 6\gamma_q$, $\gamma_q = 1/6$.

T_c (MeV)	ΔF_c (MeV)	R_c (fm)	σ (MeV/fm^2)	σ/T_c^3
150	1928.00	4.416	14.865	0.17
160	2377.00	4.488	18.069	0.17
170	2890.00	4.555	21.667	0.17
190	4104.00	4.659	30.233	0.17
210	5541.00	4.713	40.735	0.17
230	7132.00	4.703	53.461	0.17
250	8775.00	4.618	68.974	0.17

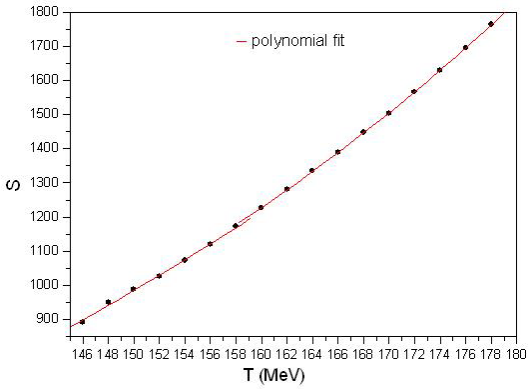

Figure 3. Variation of S with temperature T at $\gamma_g = 6\gamma_q$ and $\gamma_q = 1/6$ with the effect of curvature.

with the inclusion of curvature term in the total energy for parametrization factor $\gamma_g = 6\gamma_q$ and a very mild discontinuity in the entropy of the parametrization factor $\gamma_g = 8\gamma_q$. It happened in the first-order thermodynamic variable like entropy. The discontinuity is just of the order of one standard deviation of the entropy variable and therefore indeed claim to be a very weak transition. But second-order thermodynamic variable like specific heat indicates complete continuity in its behaviour.

6. Comparison and conclusion

We compare the free energies of Ramanathan *et al* model [4] with Shukla *et al* model. It is shown in figure 5 and free energy of Shukla model with the inclusion of curvature is shown in figure 6. In our calculation, potential model is used in the construction of density of states for quarks and gluons, which gives a very good explanation in the case of particle multiplicities. The stable droplet formation is clearly observed with the free energy with our chosen parametrizations whereas the droplet formation in free energies calculated with Shukla *et al* may be so rapid

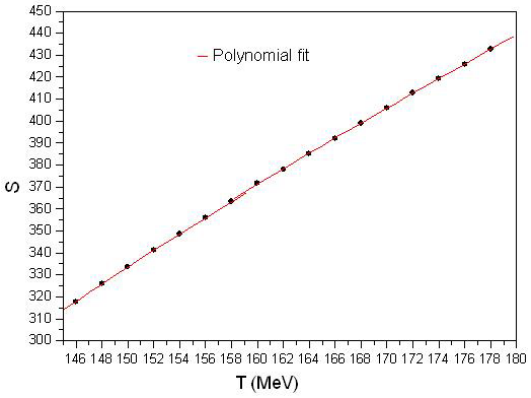

Figure 4. Variation of S with temperature T at $\gamma_g = 8\gamma_q$ and $\gamma_q = 1/6$ with the effect of curvature.

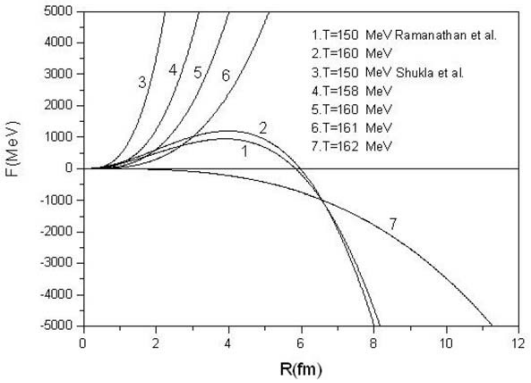

Figure 5. The comparative plot of total free energies vs. R (fm) of Ramanathan model and Shukla model for various temperatures. The free energy is contributed by u, d, s and hadrons.

that transition is directly obtained without showing any potential barrier with the increase in temperature. This may be the case when the system spontaneously passes into a QGP phase without the formation of droplets. Again we use the phase boundary separating the two phases as interfacial surface instead of MIT bag model which are used by both Kapusta *et al* and Shukla *et al*. The MIT bag model is simple; it puts all quarks and gluons as free particles inside a bag and makes the impermeable bag as the agent of confinement by ascribing a set of boundary conditions for quarks and gluons. It is fine to use the MIT model to describe the hadrons as bags of quarks, antiquarks and gluons, but to extend the idea to represent the phase boundary between the QGP droplet and the bulk hadronic medium makes one a bit uneasy. It is to remedy this rather unnatural assumption, i.e. the confining bag of the hadrons has the same property as the interface separating the two phases. Ramanathan *et al* proposed an alternative model to represent the same physical situation. Another drawback of the MIT bag

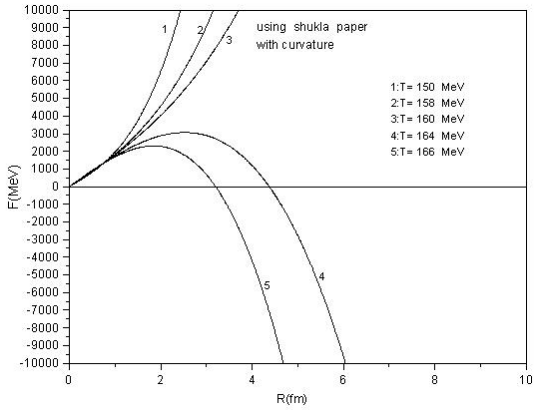

Figure 6. The total free energies vs. R (fm) of Shukla model for various temperatures with the inclusion of curvature.

model is its disagreement with 'numerical experiments' using lattice gauge pure $SU(3)$ simulation. As pointed out by Peshier *et al*, the simulation data is satisfied only by a bag pressure $P = ae - 4B/3$, with $a = 0.297$ (not $1/3$ as for MIT bag model), where e is the energy density and $B^{1/4} = 205$ MeV for the bag constant. This mismatch with lattice simulations had led earlier authors [9] and Ramanathan *et al* to abandon the MIT bag model in the context of QGP by introducing 'thermal parton masses'. Moreover, with the inclusion of curvature in their model, we can see the proper barrier amplitude in the free energy shown in figure 6 and this amplitude can be used in the calculation of surface tension. However, in this paper, correction with curvature in the free energy for the two parametrization factors are shown in figures 1 and 2. This has a good improvement in the free energy over free energy without this curvature correction. The energy at R_c is about two times the energy without the curvature. There is increase in the maximum stable droplet size too. Due to this significant effect in the amplitude of the free energy, we determine the surface tension, thermodynamic variable like entropy for these two factors $\gamma_g = 6\gamma_q$ and $\gamma_g = 8\gamma_q$. There is a significant increase in surface tension at this critical temperature with this modified free energy. This shows a good improvement over the earlier results of Ramanathan *et al* without curvature. This indicates that the leading order term of curvature has a good impact to find the surface tension. The calculated value of surface tension is found to be $0.17T_c^3$, which is two times the earlier results, that is, $0.078T_c^3$ [4] and this present value is nearly close to higher value of lattice results that is $\leq 0.24T_c^3$ [7,13]. The result is still consistent with earlier ones in terms of its constancy throughout the temperature and parametrization factors. It is in the wake of striking conformity with the results of latest lattice QCD simulations [10]. The model still shows weakly first-order phase transition at the temperature in the range (160 ± 5) MeV with the effect of curvature in the free energy and this is expected with the current feature of QGP–hadron phase transition too [1]. This transition is a mild characteristic in nature and discontinuity is found in the first-order thermodynamic variable, namely, the entropy S as shown in figures 3 and 4 and it shows continuity in the behaviour of

specific heat. It is a fact that our results of surface tension, entropy and specific heat are expected with the recent model calculations of lattice simulations in QCD phase transition [16] and, unlike a cross-over transition, overall it is a weakly first-order phase transition.

Acknowledgments

The authors thank H T Elze for fruitful discussion and his comment on this paper. We are very thankful to Dr R Ramanathan for his support in preparing this manuscript. DSG and YK are very grateful to the Department for giving them opportunities to do research here and very thankful to CSIR and Rajiv Gandhi Fellowship for financial support.

References

[1] C Y Wong, *Introduction to high-energy heavy ion collisions* (World Scientific, Singapore, 1994)
L P Csernai, *Introduction to relativistic heavy-ion collisions* (Wiley, New York, 1994)
R C Hwa, *Quark-gluon plasma* (World Scientific, Singapore, 1990)
[2] H Satz, CERN-TH-2590, 18pp (1978)
F Karsch, E Laermann, A Peikert, Ch Schmidt and S Stickan, *Nucl. Phys.* **B94**, 411 (2001)
F Karsch and H Satz, *Nucl. Phys.* **A702**, 373 (2002)
[3] L P Csernai, J I Kapusta and E Osnes, *Phys. Rev.* **D67**, 045003 (2003)
J I Kapusta, R Venugopal and A P Vischer, *Phys. Rev.* **C51**, 901 (1995)
L P Csernai and J I Kapusta, *Phys. Rev. Lett.* **69**, 737 (1992)
L P Csernai, J I Kapusta, Gy Kluge and E E Zabrodin, *Z. Phys.* **C58**, 453 (1993)
[4] R Ramanathan, Y K Mathur, K K Gupta and Agam K Jha, *Phys. Rev.* **C70**, 027903 (2004)
R Ramanathan, K K Gupta, Agam K Jha and S S Singh, *Pramana – J. Phys.* **68**, 757 (2007)
[5] B D Malhotra and R Ramanathan, *Phys. Lett.* **A108**, 153 (1985); See also ref. [3]
[6] I Mardor and B Svetitsky, *Phys. Rev.* **D44**, 878 (1991)
M G Mustafa and A Ansari, *Z. Phys.* **C57**, 51 (1993)
M G Mustafa, D K Srivastava and B Sinha, *Euro. Phys. J.* **C5**, 711 (1998)
[7] G Neergaad and J Madsen, *Phys. Rev.* **D60**, 054011 (1999)
M B Christiansen and J Madsen, *J. Phys.* **G23**, 2039 (1997)
H T Elze and W Greiner, *Phys. Lett.* **B179**, 385 (1986)
[8] A D Linde, *Nucl. Phys.* **B216**, 421 (1983)
E Fermi, *Zeit F. Physik* **48**, 73 (1928)
L H Thomas, *Proc. Camb. Phil. Soc.* **23**, 542 (1927)
H A Bethe, *Rev. Mod. Phys.* **9**, 69 (1937)
[9] A Peshier, B Kampfer, O P Pavlenko and G Soff, *Phys. Lett.* **B337**, 235 (1994)
V Goloviznin and H Satz, *Z. Phys.* **C57**, 671 (1993)
[10] Y Iwasaki, K Kanaya, L Kärkkäinen, K Rummukainen and T Yoshic, *Phys. Rev.* **D49**, 3540 (1994)
[11] H Weyl, *Nachr. Akad. Wiss Gottingen* 110 (1911)

[12] J Solfrank, P Husvinen, M Kataja, P V Ruuskanen, M Prakash and R Venugopalan, *Phys. Rev.* **C55**, 392 (1997)
C M Hung and E Shuryak, *Phys. Rev.* **C57**, 1891 (1998)
E Shuryak, *Prog. Part. Nucl. Phys.* **53**, 273 (2004)
[13] Rajiv V Gavai, *J.O. Phys.: Confer. Series* **50**, 87 (2006)
K M Udayananda, P Sethumadhavan and V M Bannur, *Phys. Rev.* **C76**, 044908 (2007)
[14] Sanjay K Ghosh, Tamal K Mukherjee, Munshi G Mustafa and Rajarshi Ray, *Phys. Rev.* **D73**, 114007 (2006)
[15] A Ali Khan *et al*, *Phys. Rev.* **D64**, 074510 (2001)
Yasumichi Aoki, Zoltan Fodor, Sandor D Katz and Kalman K Szabo, *J. High Energy Phys.* **01**, 089 (2006)
[16] Rajiv V Gavai, Sourendu Gupta and Swagato Mukherjee, *Pramana – J. Phys.* **71**, 487 (2008)
[17] P Shukla and A K Mohanty, *Phys. Rev.* **C64**, 054910 (2001)
A K Mohanty, P Shukla and M Gleiser, *Phys. Rev.* **C65**, 034908 (2002)