Int J Theor Phys (2011) 50:2609–2613
DOI 10.1007/s10773-011-0756-4

CPT Violating Neutrino Oscillation Under Planck Scale Effects

Bipin Singh Koranga · Vinod Kumar · A. Jha

Received: 9 January 2011 / Accepted: 10 March 2011 / Published online: 24 March 2011

Abstract We give a phenomenological model of CPT violating neutrino oscillation above the GUT scale. In this paper, we consider the effect of Planck scale operators on neutrino mixing. We assume that the main part of neutrino masses and mixing arise through GUT scale operators The dispersion relation for the CPT violating neutrino and anti-neutrino oscillation are discussed.

Keywords CPT violation · Planck scale

1 Introduction

The phenomenon of neutrino oscillations has been established by solar [1], atmospheric [2] reactor [3] and long baseline accelerator [4] neutrino experiments. An asymmetry between the survival probability $P(\nu_e \to \nu_\mu)$ of electron neutrinos and that $P(\bar{\nu}_e \to \bar{\nu}_e)$ of electron anti-neutrinos, which is clear signal of a CPT violation [5]. At 99% confidence level, $A_{ee} = P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_e)$ is negative with best-fit value of [6]

$$A_{ee} = P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_e) = -0.0165. \tag{1}$$

CPT violation would signal,through the CPT theorem, a violation of the Lorentz. Non violation of CPT symmetry automatically implies a non-vanishing asymmetry

$$A_{ee} = P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_e) \neq 0. \tag{2}$$

In this paper,we would explore a CPT violating neutrino oscillation probability above the GUT scale. In this model, we take the neutrino oscillation parameter above GUT scale and then corresponding dispersion relation are obtained. The outline of the article is as follows. In Sect. 2, we briefly discuss neutrino oscillation parameter due to Planck scale effects. In Sect. 3, we discuss about CPT violating neutrino oscillation due to Planck scale effects. In Sect. 4, we present our conclusions.

B.S. Koranga · V. Kumar (✉) · A. Jha
Department of Physics, Kirori Mal College, University of Delhi, Delhi 110007, India
e-mail: vinodpdu@rediffmail.com

2 Neutrino Oscillation Parameter Due to Planck Scale Effects

The neutrino mass matrix is assumed to be generated by the see saw mechanism [8, 9]. The effective gravitational interaction of neutrino with Higgs field can be expressed as $SU(2)_L \times U(1)$ invariant dimension-5 operator [7],

$$L_{grav} = \frac{\lambda_{\alpha\beta}}{M_{pl}}(\psi_{A\alpha}\epsilon\psi_C)C_{ab}^{-1}(\psi_{B\beta}\epsilon_{BD}\psi_D) + h.c. \tag{3}$$

Here and every where we use Greek indices α, β for the flavor states and Latin indices i, j, k for the mass states. In the above equation $\psi_\alpha = (\nu_\alpha, l_\alpha)$ is the lepton doublet, $\phi = (\phi^+, \phi^o)$ is the Higgs doublet and $M_{pl} = 1.2 \times 10^{19}$ GeV is the Planck mass λ is a 3×3 matrix in a flavor space with each elements $O(1)$. The Lorentz indices $a, b = 1, 2, 3, 4$ are contracted with the charge conjugation matrix C and the $SU(2)_L$ isospin indices $A, B, C, D = 1, 2$ are contracted with $\epsilon = i\sigma_2$, $\sigma_m(m = 1, 2, 3)$are the Pauli matrices. After spontaneous electroweak symmetry breaking the Lagrangian in (1) generated additional term of neutrino mass matrix

$$L_{mass} = \frac{v^2}{M_{pl}}\lambda_{\alpha\beta}\nu_\alpha C^{-1}\nu_\beta, \tag{4}$$

where $v = 174$ GeV is the *VEV* of electroweak symmetric breaking. We assume that the gravitational interaction is "flavor blind" that is $\lambda_{\alpha\beta}$ is independent of α, β indices. Thus the Planck scale contribution to the neutrino mass matrix is

$$\mu\lambda = \mu\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{5}$$

where the scale μ is

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} \text{ eV}. \tag{6}$$

We take (5) as perturbation to the main part of the neutrino mass matrix, that is generated by GUT dynamics. To calculate the effects of perturbation on neutrino observables. The calculation developed in an earlier paper [10]. A natural assumption is that unperturbed (0th order mass matrix) M is given by

$$\mathbf{M} = U^* \operatorname{diag}(M_i)U^\dagger, \tag{7}$$

where, $U_{\alpha i}$ is the usual mixing matrix and M_i, the neutrino masses is generated by Grand unified theory. Most of the parameter related to neutrino oscillation are known, the major expectation is given by the mixing elements U_{e3}. We adopt the usual parametrization.

$$\frac{|U_{e2}|}{|U_{e1}|} = \tan\theta_{12}, \tag{8}$$

$$\frac{|U_{\mu 3}|}{|U_{\tau 3}|} = \tan\theta_{23}, \tag{9}$$

$$|U_{e3}| = \sin\theta_{13}. \tag{10}$$

In term of the above mixing angles, the mixing matrix is

$$U = \mathrm{diag}(e^{if1}, e^{if2}, e^{if3})R(\theta_{23})\Delta R(\theta_{13})\Delta^* R(\theta_{12})\mathrm{diag}(e^{ia1}, e^{ia2}, 1). \tag{11}$$

The matrix $\Delta = \mathrm{diag}(e^{\frac{1\delta}{2}}, 1, e^{\frac{-i\delta}{2}})$ contains the Dirac phase. This leads to CP violation in neutrino oscillation $a1$ and $a2$ are the so called Majoring phase, which effects the neutrino less double beta decay. $f1$, $f2$ and $f3$ are usually absorbed as a part of the definition of the charge lepton field. Planck scale effects will add other contribution to the mass matrix that gives the new mixing matrix can be written as [11]

$$\begin{aligned}
U^{'} &= U(1 + i\delta\theta), \\
&\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\
&+ i \begin{pmatrix} U_{e2}\delta\theta^*_{12} + U_{e3}\delta\theta^*_{23,} & U_{e1}\delta\theta_{12} + U_{e3}\delta\theta^*_{23}, & U_{e1}\delta\theta_{13} + U_{e3}\delta\theta^*_{23} \\ U_{\mu 2}\delta\theta^*_{12} + U_{\mu 3}\delta\theta^*_{23,} & U_{\mu 1}\delta\theta_{12} + U_{\mu 3}\delta\theta^*_{23}, & U_{\mu 1}\delta\theta_{13} + U_{\mu 3}\delta\theta^*_{23} \\ U_{\tau 2}\delta\theta^*_{12} + U_{\tau 3}\delta\theta^*_{23}, & U_{\tau 1}\delta\theta_{12} + U_{\tau 3}\delta\theta^*_{23}, & U_{\tau 1}\delta\theta_{13} + U_{\tau 3}\delta\theta^*_{23} \end{pmatrix}.
\end{aligned} \tag{12}$$

Where $\delta\theta$ is a hermition matrix that is first order in μ [10]. The first order mass square difference $\Delta M^2_{ij} = M^2_i - M^2_j$, get modified [10] as

$$\Delta^{'}_{ij} = \Delta_{ij} + 2(M_i Re(m_{ii}) - M_j Re(m_{jj})), \tag{13}$$

where

$$m = \mu U^t \lambda U,$$

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6}\ \mathrm{eV}.$$

The change in the elements of the mixing matrix, which we parametrized by $\delta\theta$ [12], is given by

$$\delta\theta_{ij} = \frac{iRe(m_{jj})(M_i + M_j) - Im(m_{jj})(M_i - M_j)}{\Delta M^{'2}_{ij}}. \tag{14}$$

The above equation determine only the off diagonal elements of matrix $\delta\theta_{ij}$. The diagonal element of $\delta\theta_{ij}$ can be set to zero by phase invariance.

Using (12), we can calculate neutrino mixing angle due to Planck scale effects,

$$\frac{|U^{'}_{e2}|}{|U^{'}_{e1}|} = \tan\theta^{'}_{12}, \tag{15}$$

$$\frac{|U^{'}_{\mu 3}|}{|U^{'}_{\tau 3}|} = \tan\theta^{'}_{23}, \tag{16}$$

$$|U^{'}_{e3}| = \sin\theta^{'}_{13}. \tag{17}$$

For degenerate neutrinos, $M_3 - M_1 \cong M_3 - M_2 \gg M_2 - M_1$, because $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$. Thus, from the above set of equations, we see that $U^{'}_{e1}$ and $U^{'}_{e2}$ are much larger than $U^{'}_{e3}$, $U^{'}_{\mu 3}$ and $U^{'}_{\tau 3}$. Hence we can expect much larger change in θ_{12} compared to θ_{13} and θ_{23} [12].

3 CPT Violating Neutrino Oscillation Probability

The typical scenario of just two neutrino states of definite mass ν_1 and ν_2, the relevant unitary mixing matrix is simply a 2×2 sub matrix of the general mixing U. If we allow for non zero mass, we can assume that the state $|\nu_e\rangle$ and $\nu_\mu$mix to give new state $|\nu_1\rangle$ and $|\nu_2\rangle$ defined by

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \tag{18}$$

$|\nu_1\rangle$ and $|\nu_2\rangle$ are the mass eigenstates or the propagation eigenstates, we can write the equation of motion is

$$i \frac{d}{dt} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = H \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \tag{19}$$

Since $|\nu_1\rangle$ and $|\nu_2\rangle$ are eigenstates of the Hamiltonian, the state $|\nu_1\rangle$ and $|\nu_2\rangle$ evolves as

$$|\nu_1(t)\rangle = e^{-iE_1 t}|\nu_1(t)\rangle,$$
$$|\nu_1(t)\rangle = e^{-iE_1 t}|\nu_1(t)\rangle.$$

The probability of the in ital state ν_e oscillate in the state ν_μ at a time t is given by

$$P(\nu_e \to \nu_\mu) = \sin^2\theta \cos^2\theta \cos((E_2 - E_1)t). \tag{20}$$

As neutrino is an extremely relativistic particle, we can make the assumption $E \gg m$

$$E_i = \sqrt{p^2 + m_i^2} \backsimeq p + \frac{m^2}{2E}.$$

The survival probability can finally be simplified

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{12} \sin^2\left(\frac{1.27\Delta_{21}L}{E}\right), \tag{21}$$

where $\Delta_{21} = m_2^2 - m_1^2 > 0$ is the mass square differences. The same formula applies for anti-neutrino survival probabilities.

Due to Planck scale effects, the survival probabilities for neutrino and anti-neutrino states

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{12}^{'} \sin^2\left(\frac{1.27\Delta_{21}^{'}L}{E}\right), \tag{22}$$

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{12}^{'} \sin^2\left(\frac{1.27\Delta_{21}^{'}L}{E}\right), \tag{23}$$

result to be not identical. Due to Planck scale effects $\theta_{12}^{'}$ deviated $\pm 3°$, [12] in case of neutrino deviation is $+3°$, and for ant-neutrino deviation is $-3°$.

4 Numerical Results

We consider a degenerated neutrino Mass spectrum and take the common neutrino mass to be 2 eV, which is upper limit of tritium beta decay spectrum [12]. The contribution of the

Table 1 Survival probability for neutrino and ant-neutrino states. Input value are $\Delta_{31} = 0.002$ eV^2, $\Delta_{21} = 0.00008$ eV^2, $\theta_{12} = 34°$, $\theta_{23} = 45°$, $\theta_{13} = 0°$

$P(\nu_e \to \nu_\mu)$	$P(\bar{\nu}_e \to \bar{\nu}_e)$	$A_{ee} = P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_e)$
0.08	0.23	−0.15

term in the Planck scale correction, $\epsilon = 2(M_i Re(m_{11}) - M_j Re(m_{22}))$, can be additive or subtractive depending on the values of the phase a_1, a_2 and phase f_i. In our calculation, we used mixing angle as $\theta_{12} = 34°$, $\theta_{23} = 45°$, $\theta_{13} = 10°$. and $\delta = 0°$. We have taken $\Delta_{31} = 0.002$ eV^2 [13] and $\Delta_{21} = 0.00008$ eV^2 [14]. For simplicity, we have set the charge lepton phases $f_1 = f_2 = f_3 = 0$. Since we have set $\theta_{13} = 0$, the Dirac phase δ drops out of the 0th order mixing matrix. We assume phase of the oscillation term $(\frac{1.27\Delta'_{21}L}{E}) = \pi/2$. Due to Planck scale effects, the survival probabilities for neutrino and anti-neutrino states

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2(\theta_{12} + 3°), \tag{24}$$

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2(\theta_{12} - 3°). \tag{25}$$

In Table 1 we list the survival probability for neutrino and ant-neutrino states.

We note that the none zero value of A_{ee}, indicate that the CPT violation in neutrino sector above the GUT scale.

5 Conclusions

We assume that the main part of neutrino masses and mixing from GUT scale operator. We considered these to be 0th order quantize. The gravitational interaction of lepton field with S.M. Higgs field give rise to a $SU(2)_L \times U(1)$ invariant dimension-5 effective Lagrangian give originally by Weinberg [15]. On electroweak symmetry breaking this operators leads to additional mass terms. We considered these to be perturbation of GUT scale mass terms. We compute the first order correction to neutrino mass eigenvalue, mixing angles and calculate the modified mass square differences. The dependence in our model of $P(\nu_e \to \nu_\mu)$ and $P(\bar{\nu}_e \to \bar{\nu}_e)$ on Planck scale effects different from the standard case (compare (22), (23) with (21). We then expect the possible CPT violation above the GUT scale.

References

1. Kamiokande Collaboration, Fukuda, Y., et al.: Phys. Rev. Lett. **77** (1996)
2. Super-Kamiokande Collaboration, Ashie, Y., et al.: Phys. Rev. D **71**, 112005 (2005)
3. CHOOZ Collaboration, Apollonio, M., et al.: Phys. Lett. B **466**, 415 (1999)
4. K2K Collaboration, Ahn, M.H., et al.: Phys. Rev. D **74**, 072003 (2006)
5. Esposito, S., Salesi, G.: arXiv:0906.5542 [hep-ph]
6. Giunti, C., Laveder, M.: arXiv:0902.1992 [hep-ph]
7. Weinberg, S.: Phys. Rev. Lett. **43**, 1566 (1979)
8. Mohapatra, R.N., et al.: Phys. Rev. Lett. **44**, 912 (1980)
9. Coleman, S., Galshow, S.L.: Phys. Rev. D **59**, 116008 (1999)
10. Vissani, F., et al.: Phys. Lett. B **571**, 209 (2003)
11. Koranga, B.S., Narayan, M., Sankar, S.U.: Fizika B **18**, 219–226 (2009)
12. Koranga, B.S., Narayan, M., Sankar, S.U.: Phys. Lett. B **665**, 63 (2008)
13. Kraus, C., et al.: Eur. Phys. J **33** (2004)
14. Super-Kamiokande Collaboration, Hosaka, J., et al.: Phys. Rev. D **74**, 32002 (2006)
15. KamLAND Collaboration, Hosaka, J., et al.: Phys. Rev. Lett. **94**, 08180 (2005)