International Journal of Modern Physics A
Vol. 29, No. 20 (2014) 1450097 (7 pages)

DOI: 10.1142/S0217751X14500973

Restudy of surface tension of QGP with one-loop correction in the mean-field potential

S. Somorendro Singh
Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India
sssingh@physics.du.ac.in

K. K. Gupta
Department of Physics, Ramjas College, University of Delhi, Delhi-110007, India

A. K. Jha
Department of Physics, Kiroli Mal College, University of Delhi, Delhi-110007, India

Received 18 November 2013
Revised 25 April 2014
Accepted 13 May 2014
Published 2 July 2014

Surface tension of quark–gluon plasma (QGP) evolution with one-loop correction in the mean-field potential is studied. First, with the correction, the stable QGP droplet size decreases. Then, the value of surface tension is found to be improved and it approaches to the lattice value of surface tension $0.24T_c^3$. Moreover, the ratio of the surface tension to the cube of the critical temperature is found to increase the value in comparison to earlier studies without correction factor [R. Ramanathan, K. K. Gupta, A. K. Jha and S. S. Singh, *Pram. J. Phys.* **68**, 757 (2007)].

Keywords: Quark–gluon plasma; quark–hadron phase transition.

PACS numbers: 25.75.Ld, 12.38.Mh, 21.65.+f

1. Introduction

The study of phase transition is expected to occur when the confinement of colored quarks and gluons vanishes into colorless hadrons under the conditions of high energy density and temperature.[2–5] This process is believed to happen in the early stage of universe evolution. The process of this early stage of the universe can be replicated as a complicated phenomena in the heavy-ion collider experiments. So, the study of quark–gluon plasma (QGP) fireball in Ultra Relativistic Heavy-Ion Collisions has become an exciting field in the present day of heavy-ion collider physics,[6–10] and the experiments at RHIC and LHC operating with heavy-ion have become the frontier laboratories for exploring the QGP formation. Moreover it is believed that the phenomena of quasi-static equilibrium support in solving this complicated phenomena of QGP and its phase transition. Many other pioneers in this field have worked out the process of QGP phase transitions.[11–15]

In these works, it is explained that the energy density of the deconfined state may reach a very high value due to large fluctuations in the system. These fluctuations can be explained on the basis of nucleation theory. In the process of nucleation theory, it also calculates the critical free energy difference between the two phases with the liquid drop model and the energy difference of the two phases is given as:[16]

$$\Delta F = -\frac{4\pi}{3}R^3[P_{\text{had}}(T) - P_{q,g}(T)] + 4\pi R^2\sigma + \tau_{\text{crit}}T\ln\left[1 + \left(\frac{4\pi}{3}\right)R^3 s_{q,g}\right]. \tag{1}$$

In the expression, the pressure difference is attributed from the sum of the pressure contributed by the number of quarks and gluons whereas the hadronic pressure is contributed by the excessive number of pions available in the system. The second and third term represent surface and shape contribution in the free energy difference. In the surface energy, σ is the surface tension which support in the stability formation of QGP droplets. τ_{crit} and $s_{q,g}$ are the critical time and entropy of the system at the time of transformation from hadron to QGP state. The surface tension is calculated by minimizing the expression (1) with respect to the droplet size. It is obtained as:[17–20]

$$R_c = \frac{2\sigma}{\Delta p} \quad \text{or} \quad \sigma = \frac{3\pi\Delta F}{4\pi R_c^2}. \tag{2}$$

The last term representing shape contribution is neglected in lower order approximation because of large fluctuations in droplet shape. These calculations are done in our earlier works.[21–24] Now we extend the same calculation with the addition of one-loop change of the coupling factor in the interacting mean-field potential.[25–28] With the addition of one-loop factor in coupling value, there are changes in the free energy amplitude of QGP fireball and it affects in the stability of QGP droplet with the variation of dynamical quark and gluon flow parameter.

In this short paper, we briefly restudy free energy incorporating one-loop correction factor in the mean-field potential and calculate the surface tension of stable droplet and compare it with the earlier study.[1] This correction factor modified the density of state of the constituent particles of QGP.

2. Potential with One-Loop Correction and DOS

The density of states (DOS) of QGP droplet is determined adapting a phenomenological confining potential $V_{\text{conf}}(k)$. The potential is evaluated by looking the interactions between the constituent particles. The calculation is considered with first-order correction factor in the potential called mean-field potential in phase space. The correction is done in such a way that the expansion of strong coupling constant is perturbed with a loop correction between quark–antiquark and quark–gluons.[1,26,27] It is calculated through the Hamiltonian of system. So, the interacting mean-field potential is expressed as:

$$V_{\rm conf}(k) = (2\pi/k)\beta\alpha_s(k)T^2\left[1+\frac{\alpha_s(k)}{4\pi}a_1\right] - \frac{m_0^2}{2k}\,, \tag{3}$$

where

$$\beta = \sqrt{2}\times\sqrt{(1/\beta_g)^2+(1/\beta_q)^2}\,, \tag{4}$$

called the effective rms value of parametrization factor of $\beta_q = 1/8$ and $\beta_g = (8\text{–}10)\beta_q$. These factors determine the dynamics of QGP flow and subsequent transformation to the confining colorless hadrons. $\alpha_s(k)$ is the coupling value of quark and gluon with degree of freedom n_f,

$$\alpha_s(k) = \frac{4\pi}{(33-2n_f)\ln\left(1+\frac{k^2}{\Lambda^2}\right)} \tag{5}$$

in which QCD parameter is defined as $\Lambda = 0.15$ GeV. The coefficient a_1 is one-loop correction value in their interactions[28,29] and it is given as:

$$a_1 = 2.5833 - 0.2778n_l\,, \tag{6}$$

where n_l is considered with the number of light quark elements.[30,31]

So the DOS in phase space with one-loop correction factor in the potential is obtained through Ramanathan *et al.*[1,32] as:

$$\int \rho_{q,g}(k)dk = \frac{\nu}{\pi^2}[-V_{\rm conf}(k)]^2\frac{dV_{\rm conf}}{dk} \tag{7}$$

or,

$$\rho_{q,g}(k) = \frac{\nu}{\pi^2}\left[\frac{\beta_{q,g}^3 T^2}{2}\right]^3 g^6(k)A \tag{8}$$

where,

$$A = \left\{+\frac{\alpha_s a_1}{\pi}\right\}^2\left[\frac{(1+\alpha_s a_1/\pi)}{k^4} + \frac{2(1+2\alpha_s a_1/\pi)}{k^2(k^2+\Lambda^2)\ln\left(1+\frac{k^2}{\Lambda^2}\right)}\right], \tag{9}$$

ν is the volume occupied by the QGP and k is the relativistic four-momentum in natural units and $g^2(k) = 4\pi\alpha_s(k)$.

3. The Free Energy Evolution and Surface Tension

The QGP system is considered to be composed of light quark elements like u, d and s, in a soup of gluons and hadrons like pions. So, free-energy evolution is contributed by the sum of free energies of the constituent elements of the system. It is given as:[33–35]

$$F_i = \mp T g_i \int dk \rho_{q,g}(k) \ln\left(1 \pm e^{-(\sqrt{m_i^2+k^2})/T}\right), \tag{10}$$

in which fermions and gluon represent upper and lower index. g_i is the appropriate degeneracy factor of color and particle–antiparticle degeneracy for light element quarks and it is eight in the case of gluons.[33–35] Then, the hadronic free-energy contribution is considered from pions. It is given as:[36,37]

$$F_\pi = (3T/2\pi^2)\nu \int_0^\infty k^2 dk \ln\left(1 - e^{-\sqrt{m_\pi^2+k^2}/T}\right). \tag{11}$$

In addition to these contributed energies, there is bag energy that confine the system under the MIT bag model. Yet there are a few drawback in the numerical calculations of pressure and energy density of the Bag model. To remove this drawback, we use the interfacial energy obtained through a scalar Weyl surface in Ramanathan *et al.*[32,38] with suitable modification to take care of the hydrodynamic effects, given as:

$$F_{\text{interface}} = \frac{1}{4}\beta R^2 T^3 . \tag{12}$$

Now we consider the quark masses as $m_0 = m_d = 0$ MeV and $m_s = 0.15$ GeV for the calculation of the free energy of fermions. So we can compute the total free energy F_{total} as,

$$F_{\text{total}} = \sum_i F_i + F_\pi + F_{\text{interface}} , \tag{13}$$

where i stands for u, d and s quark and gluon. Now we evaluate surface tension of the system for two possible values of parametrization factors which produce stable droplet formation shown in Tables 1 and 2 as follows.

Table 1. For surface tension of QGP droplet at $\beta_g = 8\beta_q$, $\beta_q = 1/8$.

T_c (MeV)	ΔF_c (MeV)	R_c (fm)	σ (MeV/fm^2)	$\frac{\sigma}{T_c^3}$
150	242.28	2.57	8.785	0.10
170	263.24	2.60	9.282	0.10
190	510.02	2.61	17.861	0.10
210	678.23	2.59	24.082	0.10
230	861.58	2.55	31.683	0.10
250	1053.00	2.49	40.709	0.10
270	1244.00	2.41	51.002	0.10
290	1430.00	2.32	63.265	0.10

Table 2. For surface tension of QGP droplet at $\beta_g = 10\beta_q$, $\beta_q = 1/8$.

T_c (MeV)	ΔF_c (MeV)	R_c (fm)	σ (MeV/fm^2)	$\frac{\sigma}{T_c^3}$
150	368.79	3.16	8.817	0.10
170	609.07	3.37	12.826	0.10
190	945.24	3.56	17.846	0.10
210	1391.00	3.71	27.141	0.10
230	1955.00	3.84	31.735	0.10
250	2635.00	3.93	40.648	0.10
270	3420.00	3.99	51.338	0.10
290	4288.00	4.02	63.505	0.10

4. Result and Conclusion

The free-energy evolution with the DOS modified with one-loop correction in the interacting mean-field potential is shown in Figs. 1 and 2. With this additional correction in the potential, we obtain stable droplets for a range of quark and gluon flow parameters $\beta_q = 1/8, 8\beta_q \leq \beta_g \leq 10\beta_q$.[39] These two stable droplets formations are shown in Figs. 1 and 2. The surface tension of suitable flow parameters such as $\beta_q = 1/8$, $\beta_g = 8\beta_q$ and $\beta_g = 10\beta_q$ are evaluated through the expression (2). Now surface tension $\sigma(\mathrm{MeV/fm}^2)$ is found to be increasing function with respect to the corresponding temperature for different droplet sizes. The increment is also observed with high gluon flow parameter, $\beta_q = 1/8$, $\beta_g = 10\beta_q$ at which the stability of QGP droplet is obtained near the hadronic phase. Moreover, if we consider the ratio of surface tension to the cube of critical temperature, the result is found to be

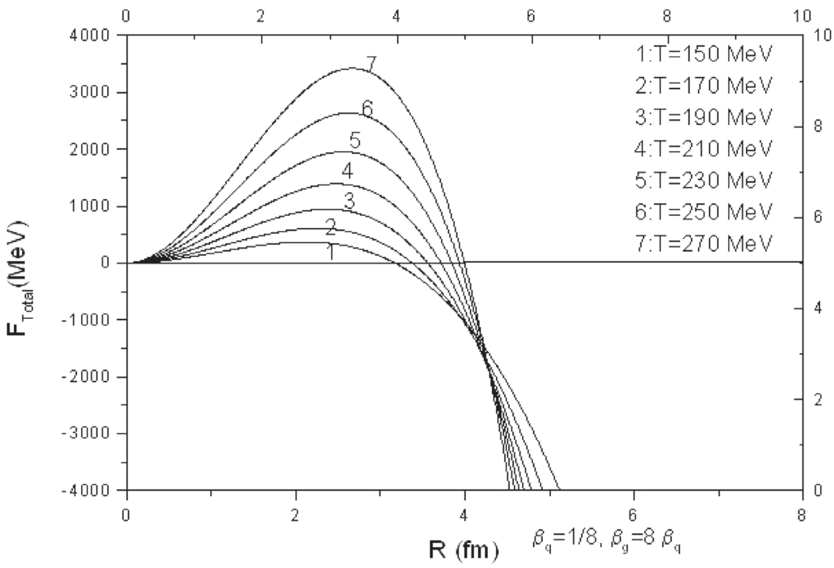

Fig. 1. Free energy F_{total} versus droplet size (R) at $\beta_g = 8\beta_q$, $\beta_q = 1/8$ for various temperature.

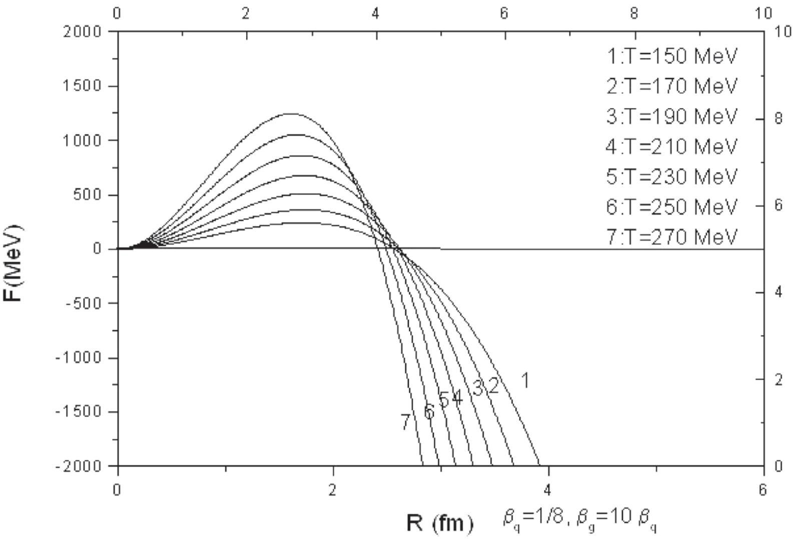

Fig. 2. Free energy F_{total} versus droplet size (R) at $\beta_g = 10\beta_q$, $\beta_q = 1/8$ for various temperature.

independent of the critical temperature and constancy is also observed in both cases of flow parameter. Thus, surface tension σ is found to be $0.10T^3$ with the correction value in the potential. It increases from the earlier value $0.078T^3$ to $0.10T^3$. This result is close to the lattice result $0.24T^3$. This means that result with the correction factor in the mean-field potential increases the value of surface tension to a very close to the lattice QCD result.[40,41]

Acknowledgments

The authors thank R. Ramanathan for his critical readings, discussions and suggestions in preparing the manuscript. We thank University Research Grant for financial support.

References

1. R. Ramanathan, K. K. Gupta, A. K. Jha and S. S. Singh, *Pram. J. Phys.* **68**, 757 (2007).
2. CP-PACS Collab. (A. Ali Khan *et al.*), *Phys. Rev. D* **63**, 034502 (2001).
3. E. Karsch, A. Peikert and E. Laermann, *Nucl. Phys. B* **605**, 579 (2001).
4. A. Tawfik, *Phys. Rev. D* **71**, 054502 (2005).
5. Z. G. Tan and A. Bonasera, *Nucl. Phys. A* **784**, 368 (2007).
6. H. Satz, CERN-TH-2590 (1978).
7. F. Karsch, E. Laermann, A. Peikert, Ch. Schmidt and S. Stickan, *Nucl. Phys. B* **94**, 411 (2001).
8. F. Karsch and H. Satz, *Nucl. Phys. A* **702**, 373 (2002).
9. E. Kolb and M. Turner, *Phys. Lett. B* **115**, 99 (1982).
10. K. Kajantie and H. Kurki-Suonio, *Phys. Rev. D* **34**, 1719 (1986).

11. L. P. Csernai, J. I. Kapusta and E. Osnes, *Phys. Rev. D* **67**, 045003 (2003).
12. M. G. Mustafa, D. K. Srivastava and B. Sinha, *Eur. Phys. J. C* **5**, 711 (1998).
13. P. Shukla and A. K. Mohanty, *Phys. Rev. C* **64**, 054910 (2001).
14. A. K. Mohanty, P. Shukla and M. Gleiser, *Phys. Rev. C* **65**, 034908 (2002).
15. A. Tawfik and T. Harko, *Phys. Rev. D* **85**, 084032 (2012).
16. J. I. Kapusta, R. Venugopalan and A. P. Vischler, *Phys. Rev. C* **51**, 901 (1995).
17. A. D. Linde, *Nucl. Phys. B* **216**, 421 (1983).
18. E. Fermi, *Z. Phys. A* **48**, 73 (1928).
19. L. H. Thomas, *Proc. Camb. Philos. Soc.* **23**, 542 (1927).
20. H. A. Bethe, *Rev. Mod. Phys.* **9**, 69 (1937).
21. C. Gale, S. Jeon and B. Schenke, arXiv:1301.5893 [nucl-th].
22. J. L. Albacete, A. Dumitru and C. Marquet, *Int. J. Mod. Phys. A* **28**, 1340010 (2013).
23. M. Luzum and H. Petersen, arXiv:1312.5503 [nucl-th].
24. A. Peshier, B. Kämpfer, O. P. Pavlenko and G. Soff, *Phys. Lett. B* **337**, 235 (1994).
25. V. Goloviznin and H. Satz, *Z. Phys. C* **57**, 671 (1993).
26. N. Brambilla, A. Pineda, J. Soto and A. Vairo, *Phys. Rev. D* **63**, 014023 (2001).
27. K. Melnikov and A. Yelkhovski, *Nucl. Phys. B* **528**, 59 (1998).
28. W. Fischler, *Nucl. Phys. B* **129**, 157 (1977).
29. A. Billoire, *Phys. Lett. B* **92**, 343 (1980).
30. A. V. Smirnov, V. A. Smirnov and M. Steinhauser, *Phys. Lett. B* **668**, 293 (2008).
31. A. V. Smirnov, V. A. Smirnov and M. Steinhauser, *Phys. Rev. Lett.* **104**, 112002 (2010).
32. R. Ramanathan, Y. Mathur, K. K. Gupta and A. K. Jha, *Phys. Rev. C* **70**, 027903 (2004).
33. G. Neergaad and J. Madsen, *Phys. Rev. D* **60**, 054011 (1999).
34. M. B. Christiansen and J. Madsen, *J. Phys. G* **23**, 2039 (1997).
35. H. T. Elze and W. Greiner, *Phys. Lett. B* **179**, 385 (1986).
36. R. Balian and C. Block, *Ann. Phys.* (*N.Y.*) **64**, 401 (1970).
37. I. Mardor and B. Svetitsky, *Phys. Rev. D* **44**, 878 (1991).
38. H. Weyl, *Nachr. Akad. Wiss. Gott., Math. Phys. Kl.* 110 (1911).
39. S. S. Singh and R. Ramanathan, axXiv:1308.3757v1 [hep-ph].
40. M. B. Christiansen and J. Madsen, *Phys. Rev. D* **53**, 5446 (1996).
41. Y. Iwasaki, K. Kanaya, L. Kärkkäinen, K. Rummukainen and T. Yoshie, *Phys. Rev. D* **49**, 3540 (1994).