MIXTURE DESIGNS BASED ON PLACKETT AND BURMAN DESIGNS

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ABSTRACT: Prescott (2000) has used augmented pair designs for the projection of response surface designs onto the mixture space and obtained orthogonally blocked designs. Singh (2003) used the Darroch and Waller quadratic model to construct orthogonal designs in two blocks for three and four components. In this paper, we have used screening designs based on Plackett and Burman Designs to obtain mixture designs by the method of projection. These mixture designs have been further compared on the basis of uniformity and optimality criteria. Orthogonal blocking of the constructed mixture designs has also been presented.

KEYWORDS: Mixture Experiments, Darroch and Waller Model, Plackett and Burman Designs, Projection Designs, Quantitative Screening Designs, Measures of Uniformity and Optimality, Orthogonal Blocking.

1. INTRODUCTION

Mixture experiments are characterized by the fact that the response(y) depends on the relative proportion of each component x_i satisfying

$$0 \le x_i \le 1,\ i = 1,2,\ldots,m \quad \text{and} \quad x_1 + x_2 + \ldots + x_m = 1 \qquad (1.1)$$

and not on the total amount of the mixture. These constraints define a (m-1) dimensional simplex.

Scheffé (1958,1963) introduced models and designs for experiments with mixtures. The linear and quadratic models given by Scheffé (1958) are as follows:

$$E(Y) = \sum_{i=1}^{m} \beta_i x_i \qquad (1.2)$$

$$E(Y) = \sum_{i=1}^{m} \beta_i x_i + \sum_{\substack{i,j=1 \\ i<j}}^{m} \beta_{ij} x_i x_j \qquad (1.3)$$

Scheffé (1963) introduced process variables for these experiments. John (1984) discussed the need of blocking for mixture experiments in the presence of process variables and constructed orthogonal block designs using Latin squares. Singh (2003) used the Darroch and Waller quadratic model to construct optimal orthogonal designs in two blocks for three and four components. Aggarwal et al. (2008) used F-squares as a base for the Darroch and Waller quadratic mixture model to construct optimal orthogonal designs in two blocks for four components.

The quadratic model by Darroch and Waller (1985) for mixture experiments is

$$E(Y) = \sum_{i=1}^{m} \beta_i x_i + \sum_{i=1}^{m} \beta_{ii} x_i^2 \qquad (1.4)$$

The model is additive in the mixture components, in the sense that it is a sum of separate functions $x_1, x_2, \ldots, x_m$. When mixture components $x_1, x_2, \ldots, x_m$ vary but the sums $x_1 + x_2 + \ldots + x_s$ and $x_{s+1} + x_{s+2} + \ldots + x_m$ ($1 \le s \le m$)are fixed, the total effect of the expected response is the sum of the effects of varying $x_1, x_2, \ldots, x_s$ and $x_{s+1}, x_{s+2}, \ldots, x_m$ separately. Such models are suitable for the designs of industrial products where mixture components have additive effects on the response function. An example where the Darroch and Waller quadratic model was more suitable than Scheffé's quadratic model was discussed by Chan et al. (1998) in the design of a solder plate used in surface- mount technology in electronic manufacturing.

In this paper, we have used Plackett and Burman Designs to construct Quantitative Screening Designs (QSDs) and then projected them to obtain mixture designs. These designs have been listed for various values of N for their uniformity and optimality measures. Section 2 contains Plackett and Burman Designs and their applications. In Section 3, we have discussed the mixture designs obtained through projection of QSDs. Section 4 compares these designs on the basis of uniformity measures. Section 5 contains the D- and A-optimality of these designs. In Section 6, we have presented orthogonal blocking of these projected designs. Section 7 gives the conclusion.

2. PLACKETT AND BURMAN DESIGNS

Plackett and Burman(1946) provided a series of two-level fractional factorial design, PB(N),for examining N-1 factors in N runs, where N is a multiple of four and N≤100. When only main effects are present, the Plackett and Burman designs (or PB designs) can estimate all of these effects. Hence these designs are also known as 'Main effect designs' or 'Screening designs'.

The PB designs hold interesting projection properties. Lin and Draper (1992) have discussed the projection property of PB designs in detail. Wang and Wu(1995) have discussed the hidden projection property of the PB designs which is a result of complex aliasing pattern between the main effects and interactions, in these designs.

These designs have been used in an enormous variety of chemical and bio chemical studies, synthetic as well as analytical. In practice, PB designs with 12 and 20 runs have been most popular.