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ORIGINAL ARTICLE

A two dissimilar unit parallel system with two phase repair by skilled and ordinary repairmen

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Abstract The paper deals with the cost-benefit and reliability analysis of a two non-identical unit parallel system with two independent repairmen—skilled and ordinary. A failed unit is first attended by skilled repairman to perform first phase repair and then it goes for second phase repair by ordinary repairman. Both types of repair discipline are FCFS. All the failure and first phase repair time distributions are exponentials with different parameters whereas second phase repair time distributions are taken general. Various important measures of system effectiveness are obtained by regenerative point technique.

Keywords Transition probability · Regenerative-point · Absorbing state · Mean time to system failure (MTSF) · Mean sojourn time · Reliability

1 Introduction

To increase the effectiveness of a system, introduction of redundancy (parallel or standby) is one of the important aspect. This goal may further be achieved after making the renewal of a failed unit. The renewal can assume various forms such as preventive maintenance and corrective-maintenance (repair). Several authors including (El-Said and EL-Sherbeny 2005; Gupta and Sharma 2007; Gupta and Shivakar 2010; Shandrasekher et al. 2004) have carried out the analysis of two and more unit system models under the assumptions that a single repairman always remains available with the system to repair a failed unit and the repaired unit becomes as good as new. Recently, El-Said and EL-Sherbeny (2010) published a paper in Sankhya analyzing a two identical unit cold standby system by regenerative point technique assuming that a single repairman completes the repair of a failed unit in two phases. They have assumed the general distributions of failure time and both phases of repair times, but the equations developed in the paper for reliability, availability and busy period are true only for exponential distributions i.e. for other than exponential distribution of failure and repair times, the results obtained by Khaled et al. are not correct. In real life situation, sometimes we may come across the situation when the repair process of a failed unit completes in different phases and for each phase repair a separate repairman is needed. A real practical situation can be visualized at any automobile service station where two types of server are available one is company trained engineer and other is local mechanic. Upon arrival of any automobile for servicing, it is first attended by company trained engineer who inspect the unit critically and identify the faults in the unit. Thereafter, he instructs the local mechanic to remove/repair the unit accordingly.

Keeping the above fact in view, in the present paper, we analyze a two non-identical unit active (parallel) redundant system model assuming that the repair of a failed unit is completed in two phases—phase-I and phase-II. In phase-I, the failed unit is first attended by skilled repairman to inspect and identify the faults and then in phase-II, the process of repair is performed by ordinary repairman. Both types of repair are performed by separate repairmen who

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are always available with the system. After completion of the repair in phase-II, unit becomes as good as new. The following useful measures of system effectiveness are obtained, by using regenerative point technique.

1. The reliability and MTSF of the system.
2. Point-wise and steady state availabilities of the system.
3. Expected up time of the system due to the operation of only one unit and operation of both the units simultaneously during time interval (0, t).
4. Expected busy period of skilled and ordinary repairmen during time (0, t).
5. Net expected profit earned by the system in the interval (0, t) and in steady state.

2 System description and assumptions

1. The system comprises of two non-identical units in parallel configuration. The operation of only one unit is sufficient to do the job.
2. Each unit of the system has two modes—Normal (N) and total failure (F). Initially both the units are operative in normal mode.
3. Two repairmen skilled and ordinary are always available with the system. In phase-I, skilled repairman inspects and identifies the faults in failed unit and give instructions of repair to ordinary repairman. In phase-II, the ordinary repairman repairs the faulty components of the failed unit as per directions of skilled repairman.
4. A failed unit is first attended by skilled repairman provided he is not busy in identifying the faults of other unit; otherwise it waits for skilled repairman. After completing the work the skilled repairman handovers the failed unit to ordinary repairman to complete the repair. As soon as the repair by ordinary repairman is completed, unit becomes as good as new.
5. The repair disciplines in phase-I and phase-II are first come first serve (FCFS).
6. All the failure and phase-I repair time distributions of both units are assumed to be exponentials with different parameters whereas the phase-II repair time distributions are taken general. Further, the random variables denoting failure times of both the units and both phases repair times of each unit are assumed to be independent and uncorrelated with each other.

3 Notations and states of the system

We define the following symbols for the states of the system:

Mode symbols:

N Normal mode
F Failure mode

Suffix symbols:

o Operative
r_1 Under phase-I repair
wr_1 Waiting for phase-I repair
r_2 Under post repair
wr_2 Waiting for phase-II repair

Using the above symbols for two non-identical units in view of the assumptions stated in Sect. 2, the possible states of the system model are shown in the transition diagram (Fig. 1). In each state, the first combination of the symbols indicates the situation of first unit and second combinations of the symbols indicate the situation of second unit. Therefore, the order of the situation of the units in each state is significant. In Fig. 1, the epochs of transitions into the states S_7 from S_3, S_8 from S_4, S_9 from S_7 and S_{10} from S_8 are non-regenerative points as at these epochs the future behavior of the system depends upon past history.

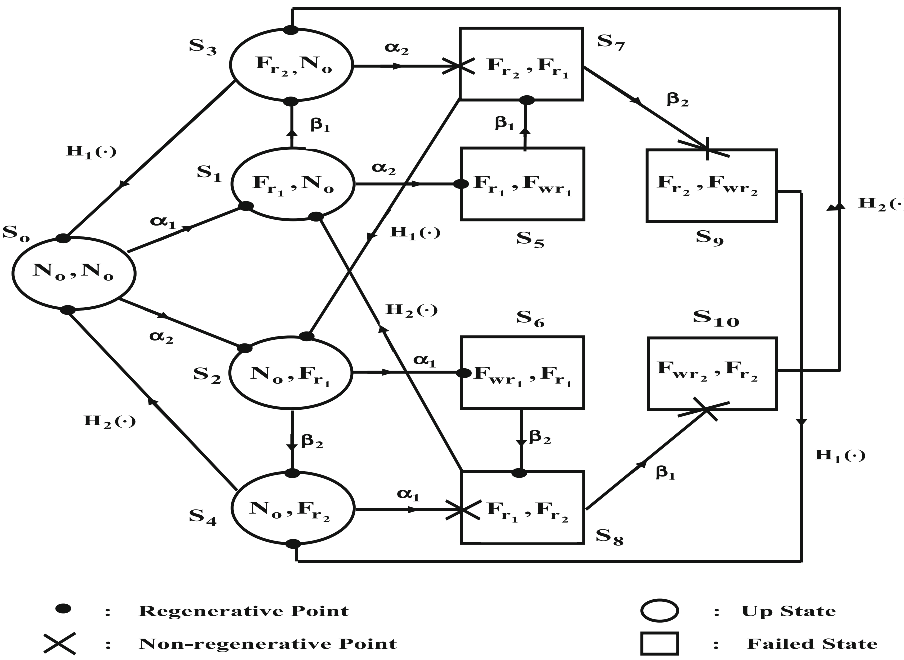
Some others notations used are as follows:

E	Set of regenerative states $\equiv \{S_0, S_1, S_2, S_3, S_4, S_7, S_8\}$
$\bar{E}$	Set of non-regenerative states $\equiv\{S_5$ to $S_{10}\}$
α_1, α_2	Constant failure rates of the first and second units
β_1, β_2	Constant repair rates of the first and second units in phase-I
$H_1(\cdot), H_2(\cdot)$	Cdf of phase-II repair time of first and second units
n_1, n_2	Mean repair time of first and second units in phase-II
$*$, $\sim$	Symbols for Laplace and Laplace Stieljes Transforms
$q_{ij}(\cdot)$	Pdf of transition time from state S_i to S_j
$q_{ij}^{(k)}(\cdot), q_{ij}^{(k,l)}(\cdot)$	Pdf of transition time from state S_i to S_j via non-regenerative state S_k and non regenerative states S_k, S_l
$p_{ij}^{(k)}, p_{ij}^{(k,l)}$	Steady state transition probabilities from state S_i to S_j via non-regenerative state S_k and non regenerative states S_k, S_l

4 Transition probabilities and mean sojourn times

The non-zero elements of one and more steps steady state transition probabilities from state S_i to S_j are as follows:

Fig. 1 Transition diagram



$$p_{01} = \frac{\alpha_1}{(\alpha_1+\alpha_2)},\ p_{02} = \frac{\alpha_2}{(\alpha_1+\alpha_2)},\ p_{13} = \frac{\beta_1}{(\beta_1+\alpha_2)},$$
$$p_{15} = \frac{\alpha_2}{(\beta_1+\alpha_2)},\ p_{24} = \frac{\beta_2}{(\beta_2+\alpha_1)}, p_{26} = \frac{\alpha_1}{(\beta_2+\alpha_1)},$$
$$p_{30} = \tilde{H}_1(\alpha_2),\ p_{32}^{(7)} = \frac{\alpha_2}{(\alpha_2-\beta_2)}\left[\tilde{H}_1(\beta_2) - \tilde{H}_1(\alpha_2)\right],$$
$$p_{40} = \tilde{H}_2(\alpha_1), p_{34}^{(7,9)} = 1 - \frac{1}{(\alpha_2-\beta_2)}$$
$$\left[\alpha_2\tilde{H}_1(\beta_2) - \beta_2\tilde{H}_1(\alpha_2)\right],$$
$$p_{41}^{(8)} = \frac{\alpha_1}{(\alpha_1-\beta_1)}\left[\tilde{H}_2(\beta_1) - \tilde{H}_2(\alpha_1)\right],$$
$$p_{43}^{(8,10)} = 1 - \frac{1}{(\alpha_1-\beta_1)}\left[\alpha_1\tilde{H}_2(\beta_1) - \beta_1\tilde{H}_2(\alpha_1)\right],$$
$$p_{72} = \tilde{H}_1(\beta_2),\ p_{74}^{(9)} = 1 - \tilde{H}_1(\beta_2), p_{81} = \tilde{H}_2(\beta_1),$$
$$p_{83}^{(10)} = 1 - \tilde{H}_2(\beta_1)$$

We observe that the following relations hold:

$$p_{01} + p_{02} = 1,\ p_{13} + p_{15} = 1,\ p_{24} + p_{26} = 1,\ p_{30} + p_{32}^{(7)} + p_{34}^{(7,9)} = 1,$$
$$p_{40} + p_{41}^{(8)} + p_{43}^{(8,10)} = 1,\ p_{72} + p_{74}^{(9)} = 1,\ p_{81} + p_{83}^{(10)} = 1,\ p_{57} = p_{68} = 1 \quad (1-8)$$

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i. In particular,

$$\psi_0 = \int e^{-(\alpha_1+\alpha_2)t}dt = 1/(\alpha_1+\alpha_2) \quad (9)$$

Similarly,

$$\psi_1 = 1/(\alpha_2+\beta_1),\ \psi_2 = 1/(\alpha_1+\beta_2),$$
$$\psi_3 = \left[1 - \tilde{H}_1(\alpha_2)\right]/\alpha_2,\ \psi_4 = \left[1 - \tilde{H}_2(\alpha_1)\right]/\alpha_1$$
$$\psi_5 = 1/\beta_1,\ \psi_6 = 1/\beta_2,\ \psi_7 = \left[1 - \tilde{H}_1(\beta_2)\right]/\beta_2,$$
$$\psi_8 = \left[1 - \tilde{H}_2(\beta_1)\right]/\beta_1 \quad (10-17)$$

5 Methodology for developing equations

In order to obtain various interesting measures of system effectiveness a number of techniques are available such as semi-markov process (Agarwal 1985; Kumar et al. 1984), supplementary variable technique (Gupta and Chaudhary 1996; Zhang 1996) and regenerative-point technique (Gupta and Sharma 2007; Gupta and Shivakar 2010). The present study deals with the technique of regenerative point as it is easy to handle the problems when behavior of the system at same epochs of entrance into the states is non-Markovian. We develop the recurrence relations for reliability, availability and busy period of repairman as follows:

5.1 Reliability of the system

Here we define $R_i(t)$ as the probability that the system does not fail during time interval (0, t) when it initially starts from up state $S_i \in E$. To obtain it we regard the failed states S_5 to S_{10} as absorbing. Now, the expressions for $R_i(t)$; i = 0, 1,2,3,4; we have the following set of integral equations:

$R_0(t)$ = Probability that system sojourns in state S_0 up to time t + $\sum_j \int_0^t$ Probability that system transits from state S_0 to S_j during (u, u + du) and then starting from S_j, the system does not fail during the remaining time (t−u); j = 1, 2

$$= e^{-(\alpha_1+\alpha_2)t} + \int_0^t q_{01}(u)du R_1(t-u) + \int_0^t q_{02}(u)du R_2(t-u) = Z_0(t) + q_{01}(t)© R_1(t) + q_{02}(t)© R_2(t)$$

Similarly,

$$\begin{aligned} R_1(t) &= Z_1(t) + q_{13}(t)© R_3(t) \\ R_2(t) &= Z_2(t) + q_{24}(t)© R_4(t) \\ R_3(t) &= Z_3(t) + q_{30}(t)© R_0(t) \\ R_4(t) &= Z_4(t) + q_{40}(t)© R_0(t) \end{aligned} \qquad (18-22)$$

where $Z_1(t) = e^{-(\alpha_2+\beta_1)t}$, $Z_2(t) = e^{-(\alpha_1+\beta_2)t}$, $Z_3(t) = e^{-\alpha_2 t}\bar{H}_1(t)$, $Z_4(t) = e^{-\alpha_1 t}\bar{H}_2(t)$

5.2 Availability of the system

Let $A_i^1(t)$ and $A_i^2(t)$ be the probabilities that the system is operative at epoch t due to the operation of either of the two units and simultaneous operation of both the units respectively when system initially starts from $S_i \in E$. By simple probabilistic arguments, as in case of reliability we have the following recursive relations for $A_i^j(t)$; i = 0 to 8 and j = 1,2.

$$\begin{aligned} A_0^j(t) &= (1-\delta)Z_0(t) + q_{01}(t)© A_1^j(t) + q_{02}(t)© A_2^j(t) \\ A_1^j(t) &= \delta Z_1(t) + q_{13}(t)© A_3^j(t) + q_{15}(t)© A_5^j(t) \\ A_2^j(t) &= \delta Z_2(t) + q_{24}(t)© A_4^j(t) + q_{26}(t)© A_6^j(t) \\ A_3^j(t) &= \delta Z_3(t) + q_{30}(t)© A_0^j(t) + q_{32}^{(7)}(t)© A_2^j(t) \\ &\quad + q_{34}^{(7,9)}(t)© A_4^j(t) \\ A_4^j(t) &= \delta Z_4(t) + q_{40}(t)© A_0^j(t) + q_{41}^{(8)}(t)© A_1^j(t) \\ &\quad + q_{43}^{(8,10)}(t)© A_3^j(t) \\ A_5^j(t) &= q_{57}(t)© A_7^j(t) \\ A_6^j(t) &= q_{68}(t)© A_8^j(t) \\ A_7^j(t) &= q_{72}(t)© A_2^j(t) + q_{74}^{(9)}(t)© A_4^j(t) \\ A_8^j(t) &= q_{81}(t)© A_1^j(t) + q_{83}^{(10)}(t)© A_3^j(t) \end{aligned} \qquad (23-31)$$

where

δ = 1 and 0 respectively for j = 1 and 2. The values of $Z_i(t)$; i = 1,2,3,4 are same as given above in Sect. 5.1.

5.3 Busy period of repairmen

Let $B_i^1(t)$ and $B_i^2(t)$ be the respective probabilities that the skilled and ordinary repairmen are busy in phase-I and phase-II repair at time t, when system initially starts from $S_i \in E$. Here by using the basic probabilistic reasoning we have the following relations for $B_i^j(t)$; i = 0 to 8 and j = 1, 2

$$\begin{aligned} B_0^j(t) &= q_{01}(t)© B_1^j(t) + q_{02}(t)© B_2^j(t) \\ B_1^j(t) &= \delta Z_1(t) + q_{13}(t)© B_3^j(t) + q_{15}(t)© B_5^j(t) \\ B_2^j(t) &= \delta Z_2(t) + q_{24}(t)© B_4^j(t) + q_{26}(t)© B_6^j(t) \\ B_3^j(t) &= (1-\delta)\bar{H}_1(t) + q_{30}(t)© B_0^j(t) + q_{32}^{(7)}(t)© B_2^j(t) \\ &\quad + q_{34}^{(7,9)}(t)© B_4^j(t) \\ B_4^j(t) &= (1-\delta)\bar{H}_2(t) + q_{40}(t)© B_0^j(t) + q_{41}^{(8)}(t)© B_1^j(t) \\ &\quad + q_{43}^{(8,10)}(t)© B_3^j(t) \\ B_5^j(t) &= \delta\, e^{-\beta_1 t} + q_{57}(t)© B_7^j(t) \\ B_6^j(t) &= \delta\, e^{-\beta_2 t} + q_{68}(t)© B_8^j(t) \\ B_7^j(t) &= \delta Z_7(t) + (1-\delta)\bar{H}_1(t) + q_{72}(t)© B_2^j(t) \\ &\quad + q_{74}^{(9)}(t)© B_4^j(t) \\ B_8^j(t) &= \delta Z_8(t) + (1-\delta)\bar{H}_2(t) + q_{81}(t)© B_1^j(t) \\ &\quad + q_{83}^{(10)}(t)© B_3^j(t) \end{aligned} \qquad (32-40)$$

where $Z_1(t)$ and $Z_2(t)$ have the same values as in Sect. 5.1 and $Z_7(t) = e^{-\beta_2 t}\bar{H}_1(t)$, $Z_8(t) = e^{-\beta_1 t}\bar{H}_2(t)$. Also, δ = 1 and 0 respectively for j = 1 and 2.

6 Analysis of characteristics

6.1 Reliability and MTSF

Taking Laplace Transforms of relations (18–22), we get simple algebraic equations in $R_i^*(s)$; i = 0 to 4. Upon solving these equations for $R_0^*(s)$, we get

$$R_0^*(s) = N_1(s)/D_1(s) \qquad (41)$$

where

$$\begin{aligned} N_1(s) &= Z_0^* + q_{01}^*\left(Z_1^* + q_{13}^* Z_3^*\right) + q_{02}^*\left(Z_2^* + q_{24}^* Z_4^*\right) \\ D_1(s) &= 1 - q_{01}^* q_{13}^* q_{30}^* - q_{02}^* q_{24}^* q_{40}^* \end{aligned}$$

Here, $q_{ij}^* = q_{ij}^*(s)$ is the L.T. of transition time pdf from state S_i to S_j i.e. $q_{ij}(t)$ and $Z_i^* = Z_i^*(s)$, (i = 0 to 4) are the L. T. of

$$\begin{aligned} &Z_0(t) = e^{-(\alpha_1+\alpha_2)t},\ Z_1(t) = e^{-(\alpha_2+\beta_1)t},\ Z_2(t) = e^{-(\alpha_1+\beta_2)t} \\ &Z_3(t) = e^{-\alpha_2 t}\overline{H}_1(t),\ Z_4(t) = e^{-\alpha_1 t}\overline{H}_2(t) \end{aligned}$$

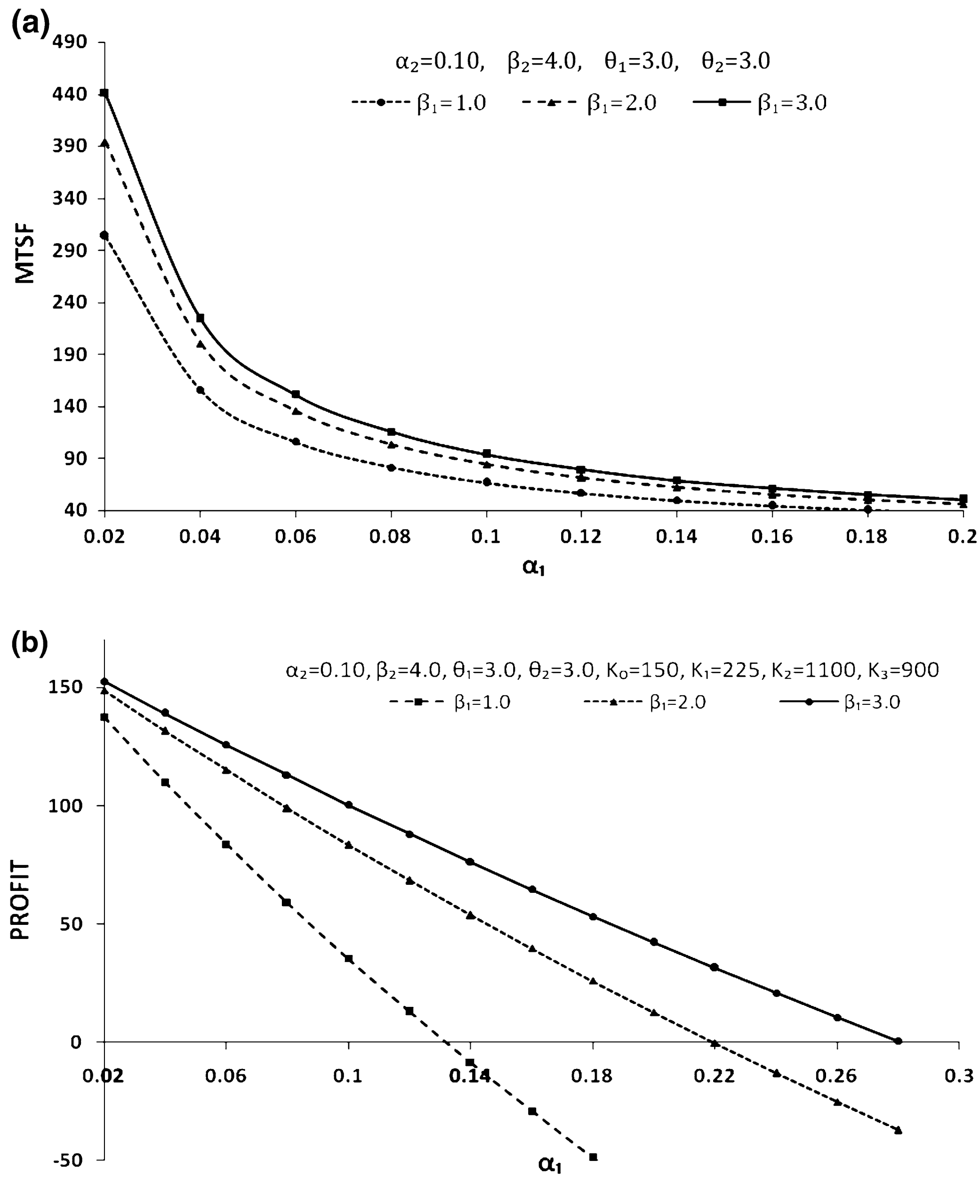


Fig. 2 Behaviour of MTSF and Profit w.r.t. α_1 for different values of β_1 for case-I

The expression of mean time to system failure is given by

$$E(T_0) = \lim_{s\to 0} R_0^*(s) = \frac{\psi_0 + p_{01}(\psi_1 + p_{13}\psi_3) + p_{02}(\psi_2 + p_{24}\psi_4)}{1 - p_{01}p_{13}p_{30} - p_{02}p_{24}p_{40}} \quad (42)$$

6.2 Availability analysis

Taking Laplace Transforms of relations (23–31) one can get the simple algebraic equations in $A_i^*(s)$; $i = 0$ to 8. Upon solving these equations we can get the values of $A_0^1(t)$and $A_0^2(t)$in terms of their L.T. i.e. $A_0^{1*}(s)$ and $A_0^{2*}(s)$. The steady-state availabilities of the system are given by

$$A_0^1 = \lim_{s\to 0} s\,A_0^{1*}(s) = (U_1\psi_1 + U_2\psi_2 + U_3\psi_3 + U_4\psi_4)/D_2 \quad (43)$$

$$A_0^2 = \lim_{s\to 0} s\,A_0^{2*}(s) = U_0\psi_0/D_2 \quad (44)$$

where

$$\begin{aligned} U_0 = {} & 1 - p_{34}^{(7,9)}p_{43}^{(8,10)} - p_{43}^{(8,10)}p_{32}^{(7)}p_{24} - p_{41}^{(8)}p_{13} \\ & \times \left(p_{34}^{(7,9)} + p_{24}p_{32}^{(7)}\right) + p_{41}^{(8)}p_{15}\left(p_{74}^{(9)} + p_{24}p_{72}\right) \\ & + p_{26}p_{15}p_{41}^{(8)}p_{83}^{(10)}\left(p_{32}^{(7)}p_{74}^{(9)} - p_{34}^{(7,9)}p_{72}\right) \\ & - p_{32}^{(7)}p_{26}p_{15}p_{81}p_{74}^{(9)}p_{43}^{(8,10)} - p_{26}p_{15}p_{81}p_{72} \\ & \times \left(1 - p_{34}^{(7,9)}p_{43}^{(8,10)}\right) - p_{32}^{(7)}p_{26}p_{83}^{(10)} - p_{13}p_{32}^{(7)}p_{26}p_{81} \end{aligned}$$

$$\begin{aligned}U_1 =& p_{01}\left[1-p_{34}^{(7,9)}p_{43}^{(8,10)}-p_{43}^{(8,10)}p_{32}^{(7)}p_{24}-p_{32}^{(7)}p_{26}p_{83}^{(10)}\right]\\ &+p_{02}\left[p_{24}p_{41}^{(8)}+p_{26}p_{34}^{(7,9)}p_{41}^{(8)}p_{83}^{(10)}+p_{26}p_{81}\left(1-p_{34}^{(7,9)}p_{43}^{(8,10)}\right)\right]\end{aligned}$$

$$\begin{aligned}U_2 =& p_{01}\left[p_{32}^{(7)}p_{13}+p_{15}p_{74}^{(9)}p_{32}^{(7)}p_{43}^{(8,10)}+p_{15}p_{72}\left(1-p_{34}^{(7,9)}p_{43}^{(8,10)}\right)\right]\\ &+p_{02}\left[1-p_{34}^{(7,9)}p_{43}^{(8,10)}-p_{13}p_{34}^{(7,9)}p_{41}^{(8)}-p_{15}p_{74}^{(9)}p_{41}^{(8)}\right]\end{aligned}$$

$$\begin{aligned}U_3 =& p_{01}\left[p_{13}+p_{15}p_{43}^{(8,10)}p_{74}^{(9)}+p_{43}^{(8,10)}p_{24}p_{72}p_{15}+p_{72}p_{15}p_{26}p_{83}^{(10)}\right]\\ &+p_{02}\left[p_{24}\left(p_{43}^{(8,10)}+p_{13}p_{41}^{(8)}\right)\right.\\ &\left.+p_{26}\left(p_{83}^{(10)}-p_{41}^{(8)}p_{83}^{(10)}p_{15}p_{74}^{(9)}-p_{15}p_{15}p_{74}^{(9)}p_{43}^{(8,10)}\right)\right]\end{aligned}$$

$$\begin{aligned}U_4 =& p_{01}\left[p_{13}\left(p_{34}^{(7,9)}+p_{32}^{(7)}p_{24}\right)+p_{15}\left(p_{74}^{(9)}+p_{72}p_{24}\right)\right.\\ &\left.-p_{26}p_{15}p_{83}^{(10)}\left(p_{32}^{(7)}p_{74}^{(9)}-p_{34}^{(7,9)}p_{72}\right)\right]\\ &+p_{02}\left[p_{24}+p_{26}p_{83}^{(10)}p_{34}^{(7,9)}+p_{81}p_{26}\left(p_{34}^{(7,9)}p_{13}+p_{15}p_{74}^{(9)}\right)\right]\end{aligned}$$

and $D_2 = U_0\psi_0 + U_1(\psi_1 + p_{15}\psi_5) + U_2(\psi_2 + p_{26}\psi_6) + (U_3 + p_{15}U_1)n_1 + (U_4 + p_{26}U_2)n_2$

The expected up (operative) time of the system during (0, t) due to the operation of either unit-1 or unit-2 and simultaneous operation of both the units are respectively given by

$$\mu_{up}^1(t) = \int_0^t A_0^1(u)du \text{ and } \mu_{up}^2(t) = \int_0^t A_0^2(u)du \quad (45)$$

so that

$$\mu_{up}^{1*}(s) = A_0^{1*}(s)/s \text{ and } \mu_{up}^{2*}(s) = A_0^{2*}(s)/s \quad (46)$$

6.3 Busy period analysis

Taking Laplace Transforms of relations (32–40) and solving the resulting set of algebraic equations, we can obtain the values of $B_0^1(t)$ and $B_0^2(t)$ in terms of their Laplace Transforms i.e. $B_0^{1*}(s)$ and $B_0^{2*}(s)$.

The steady-state probabilities that the skilled and ordinary repairman will be busy in phase-I and phase-II repair of a failed unit are respectively given by

$$B_0^1 = \lim_{s\to 0} s B_0^{1*}(s) = \left[\{\psi_1 + p_{15}(\psi_5+\psi_7)\}U_1 + \{\psi_2 + p_{26}(\psi_6+\psi_8)\}U_2\right] \Big/ D_2 \quad (47)$$

and

$$\begin{aligned}B_0^2 &= \lim_{s\to 0} s B_0^{1*}(s)\\ &= [n_1(U_3 + p_{15}U_1) + n_2(U_4 + p_{26}U_2)]/D_2 \quad (48)\end{aligned}$$

The expected busy period of skilled and ordinary repairman in phase-I and phase-II repair of a failed unit during time (0, t) are respectively given by

$$\mu_b^1(t) = \int_0^t B_0^1(u)du \text{ and } \mu_b^2(t) = \int_0^t B_0^2(u)du \quad (49)$$

so that

$$\mu_b^{1*}(s) = B_0^{1*}(s)/s \text{ and } \mu_b^{2*}(s) = B_0^{2*}(s)/s \quad (50)$$

7 Profit function analysis

We are now in the position to obtain the profit function by considering mean up time of the system during (0, t) and expected busy period of both types of repairmen during (0, t).

Let us suppose,

- K_0 is revenue per unit up-time due to the operation of single unit,
- K_1 is revenue per unit up-time due to the operation of both the units simultaneously,
- K_2 is payment to skilled repairman per unit time when he is busy in inspection and, identify the faults in failed unit and
- K_3 is payment to the ordinary repairman per unit time when he is busy in repair as per guide lines of skilled repairman.

Then the net expected profit incurred in time interval (0, t) is

$$P(t) = K_0\mu_{up}^1(t) + K_1\mu_{up}^2(t) - K_2\mu_b^1(t) - K_3\mu_b^2(t) \quad (51)$$

The expected profit per unit time in steady state is given by

$$\begin{aligned}P &= \lim_{t\to\infty}\frac{P(t)}{t} = \lim_{s\to 0} s^2P^*(s)\\ &= K_0A_0^1 + K_1A_0^2 - K_2B_0^1 - K_3B_0^2 \quad (52)\end{aligned}$$

8 Case studies

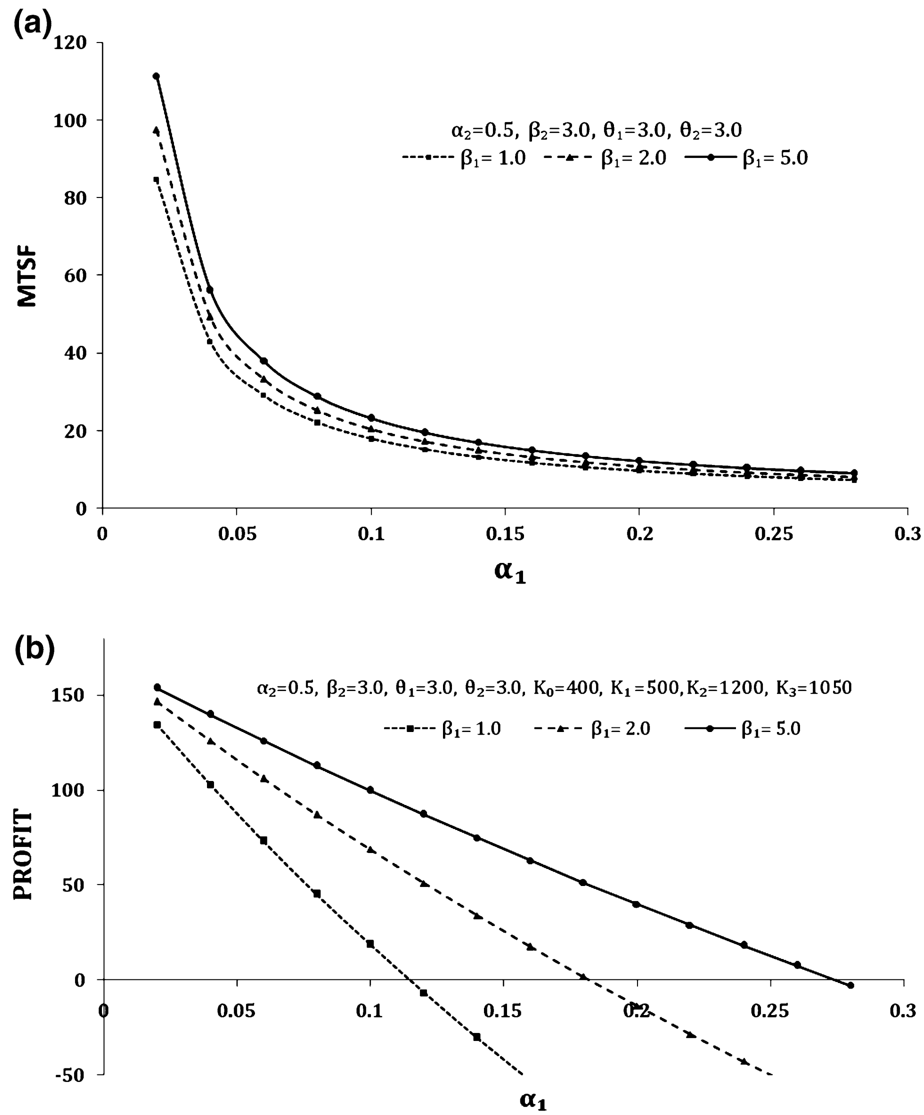
The results derived have wide applicability for various distributions of phase-II repair such as exponential, gamma, Rayleigh, weibull, lognormal, inverse-gaussion, lindley, erlangian etc. As an instance, we consider the following two case studies:

Case 1: When, phase-II repair times are also exponentials i.e.

$H_1(t) = 1 - \exp(-\theta_1 t)$ and $H_2(t) = 1 - \exp(-\theta_2 t)$

Then the changed transition probabilities and mean sojourn times of Sect. 4 are as follows:

Fig. 3 Behaviour of MTSF and Profit w.r.t α_1 for different values of β_1 for case-II



$$p_{30} = \frac{\theta_1}{(\theta_1 + \alpha_2)},\ p_{32}^{(7)} = \frac{\theta_1\alpha_2}{(\theta_1 + \beta_2)(\theta_1 + \alpha_2)},$$

$$p_{34}^{(7,9)} = \frac{\beta_1\alpha_2}{(\theta_1 + \beta_2)(\theta_1 + \alpha_2)}$$

$$p_{40} = \frac{\theta_2}{(\theta_2 + \alpha_1)},\ p_{41}^{(8)} = \frac{\theta_2\alpha_1}{(\theta_2 + \beta_1)(\theta_2 + \alpha_1)},$$

$$p_{43}^{(8,10)} = \frac{\beta_2\alpha_1}{(\theta_2 + \beta_1)(\theta_2 + \alpha_1)}$$

$$p_{72} = \frac{\theta_1}{(\theta_1 + \beta_2)},\ p_{74}^{(9)} = \frac{\beta_2}{(\theta_1 + \beta_2)},\ p_{81} = \frac{\theta_2}{(\theta_2 + \beta_1)},$$

$$p_{83}^{(10)} = \frac{\beta_1}{(\theta_2 + \beta_1)}$$

And mean sojourn times are as follows:

$$\psi_3 = 1/(\theta_1 + \alpha_2),\ \psi_4 = 1/(\theta_2 + \alpha_1),\ \psi_7 = 1/(\theta_1 + \beta_2)$$

$$\psi_8 = 1/(\theta_2 + \beta_1),\ n_1 = 1/\theta_1,\ n_2 = 1/\theta_2$$

Case 2: When, phase-II repair times follow Lindley distribution i.e.

$$h_1(t) = \frac{\theta_1^2(1+t)e^{-\theta_1 t}}{(1+\theta_1)} \text{ and } h_2(t) = \frac{\theta_2^2(1+t)e^{-\theta_2 t}}{(1+\theta_2)}$$

The Lindley distribution has the decreasing hazard rate i.e. phase-II repair rates of unit-1 and unit-2 will be respectively

$$r_1(t) = \frac{\theta_1^2(1+t)}{(1+\theta_1+\theta_1 t)} \text{ and } r_2(t) = \frac{\theta_2^2(1+t)}{(1+\theta_2+\theta_2 t)}$$

Then the changed transition probabilities and mean sojourn times of Sect. 4 are as follows:

$$p_{30} = \tilde{H}_1(\alpha_2) = \frac{(\alpha_2 + \theta_1 + 1)\theta_1^2}{(1+\theta_1)(\alpha_2+\theta_1)^2},$$

$$p_{40} = \tilde{H}_2(\alpha_1) = \frac{(\alpha_1 + \theta_2 + 1)\theta_2^2}{(1+\theta_2)(\alpha_1+\theta_2)^2}$$

$$p_{72} = \tilde{H}_1(\beta_2) = \frac{(\beta_2 + \theta_1 + 1)\theta_1^2}{(1+\theta_1)(\beta_2+\theta_1)^2},\ p_{74}^{(9)} = 1 - \tilde{H}_1(\beta_2)$$

$$p_{81} = \tilde{H}_2(\beta_1) = \frac{(\beta_1 + \theta_2 + 1)\theta_2^2}{(1+\theta_2)(\beta_1+\theta_2)^2},\ p_{83}^{(10)} = 1 - \tilde{H}_2(\beta_1)$$

$$p_{32}^{(7)} = \frac{\alpha_2}{(\alpha_2 - \beta_2)}\left[\tilde{H}_1(\beta_2) - \tilde{H}_1(\alpha_2)\right],$$

$$p_{34}^{(7,9)} = 1 - \frac{1}{(\alpha_2 - \beta_2)}\left[\alpha_2\tilde{H}_1(\beta_2) - \beta_2\tilde{H}_1(\alpha_2)\right]$$

$$p_{41}^{(8)} = \frac{\alpha_1}{(\alpha_1 - \beta_1)}\left[\tilde{H}_2(\beta_1) - \tilde{H}_2(\alpha_1)\right],$$

$$p_{43}^{(8,10)} = 1 - \frac{1}{(\alpha_1 - \beta_1)}\left[\alpha_1\tilde{H}_2(\beta_1) - \beta_1\tilde{H}_2(\alpha_1)\right]$$

And mean sojourn times are as follows:

$$\psi_3 = [1 - p_{30}]/\alpha_2,\ \psi_4 = [1 - p_{40}]/\alpha_1,\ \psi_7 = [1 - p_{72}]/\beta_2$$

$$\psi_8 = [1 - p_{81}]/\beta_1,\ n_1 = \frac{(\theta_1 + 2)}{\theta_1(1+\theta_1)},\ n_2 = \frac{(\theta_2 + 2)}{\theta_2(1+\theta_2)}$$

9 Graphical studies of characteristics and conclusions

To study the system behavior through graphs, we plot the curves for MTSF and profit function for both the particular cases.

In Case 1, when the phase-II repair time follows exponential distribution, Fig. 2a, b depicts the variation in MTSF and steady state profit with respect to failure rate α_1 for three different values of repair rate β_1(=1.0, 2.0. 3.0). We may clearly observe that MTSF decrease with the increase in α_1 and increases with increase in repair rate β_1. The same trends are observed for the graph of steady state profit in respect of α_1 and β_1. Further from Fig. 2b we observe that system is profitable only if α_1 is less than 0.14, 0.22 and 0.28 for $\beta_1 = 1.0$, 2.0 and 3.0 respectively for fixed values of $\alpha_2 = 0.10$, $\beta_2 = 4.0$, $\theta_1 = 3.0$, $\theta_2 = 3.0$, $K_0 = 150$, $K_1 = 225$, $K_2 = 1{,}100$, $K_3 = 900$.

In Case 2, when the phase-II repair time follows lindley distribution, Fig. 3a, b depicts the variation in MTSF and steady state profit with respect to parameter α_1 for three different values of repair rate β_1(=1.0, 2.0. 5.0). We may clearly observe that MTSF decrease with the increase in α_1 and increases with increase in β_1. The similar trends are observed for the graph of steady state profit in respect of α_1 and β_1. Further from Fig. 3b we observe that system is profitable only if α_1 is less than 0.12, 0.18 and 0.28 for $\beta_1 = 1.0$, 2.0 and 5.0 respectively for fixed values of $\alpha_2 = 0.5$, $\beta_2 = 3.0$, $\theta_1 = 3.0$, $\theta_2 = 3.0$, $K_0 = 400$, $K_1 = 500$, $K_2 = 1{,}200$, $K_3 = 1{,}050$.

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