

# Relationship between Structure, Entropy, and Diffusivity in Water and Water-Like Liquids

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Anomalous behavior of the excess entropy ( $\Delta S_e$ ) and the associated scaling relationship with diffusivity are compared in liquids with very different underlying interactions but similar water-like anomalies: water (SIP<sup>1</sup>) and TIP<sup>2</sup>(P) models, tetrahedral ionic melts (SiO<sub>2</sub> and BeF<sub>4</sub>), and a fluid with core-softened two-scale rump (TSR<sup>3</sup>) interactions. We demonstrate the presence of an excess entropy anomaly in the two water models. Using length and energy scales appropriate for onset of anomalous behavior, we show the density range of the excess entropy anomaly to be much narrower in water than in some melts or the 2SRP fluid. While the reduced diffusivities ( $D^*$ ) conform to the excess entropy scaling relation,  $D^* = A \exp(\beta \Delta S_e)$ , in all the systems (Montroll, E. Phys. Rev. A 1977, 15, 2025), the experimental scaling parameter  $\beta$  shows a small monotonic dependence in the case of water. Replacing  $\beta$  by pair correlation based approximations accentuates the monotonic dependence of the diffusivity scaling. Isobars with similar diffusivity scaling parameters are shown to have the temperature dependence of the corresponding entropic contribution. The relationships between diffusivity, excess entropy, and pair correlation approximations to the excess entropy are very similar in all the tetrahedral liquids.

## 1. Introduction

Water displays a number of thermodynamic and kinetic anomalies when compared to simple liquids.<sup>1</sup> The density anomaly, corresponding to a negative isobaric thermal expansion coefficient ( $\alpha_{\text{th}}$ ), is the best known of these unusual properties of water and is observed for state points lying within an approximately parabolic boundary defined by the locus of zero-pressure of vacuum density (TMD) for which  $\alpha_{\text{th}} = 0$ . The density anomaly implies the presence of other thermodynamic anomalies, such as those associated with the isobaric heat capacity ( $C_p$ ) and the isothermal compressibility ( $\kappa_T$ ). The kinetic anomalies of water are associated with an increase in molecular mobility at isothermal compression measured in terms of diffusivity, orientational relaxation times, or viscosity. A number of network-forming tetrahedral melts with local tetrahedral order have been shown to possess water-like anomalies, most notably, Al<sub>2</sub>O<sub>3</sub> compounds such as SiO<sub>2</sub> and BeF<sub>4</sub>, and the liquid phase of elements such as silicon and sulfur.<sup>2</sup> More recently, water-like anomalies have been demonstrated in mesoscopic liquids with core-softened effective interactions or atomistic particle interactions.<sup>3–5</sup> Despite a very diverse set of underlying interactions, liquids with water-like anomalies are found to have essentially conformal liquid-state “phase diagrams,” with respect to the structure of anomalous regimes of density, diffusivity, and structural order.

The similarity in the phase diagrams of water-like liquids reflects similar structure–entropy–diffusivity relationships that can be conveniently analyzed in terms of the excess entropy,  $\Delta S_e$ , defined as the difference between the total thermodynamic entropy ( $S$ ) and the corresponding ideal gas entropy ( $S_0$ ) at the

same temperature and density. A necessary condition for a fluid to show water-like thermodynamic and transport anomalies is the existence of an excess entropy anomaly, corresponding to a rise in excess entropy,  $\Delta S_e$ , on isothermal compression ( $\rho_{\text{th}} > \rho_{\text{eq}} > \rho_{\text{cr}}^{(1,2)}$ —isophs with water-like anomalies display distinct forms of local order or length scales in the low- and high-density regimes; competition between the two types of local order results in a rise in excess entropy at intermediate densities). Most liquids, including anomalous ones, obey approximate excess-entropy-scaling relationships for transport properties of the form  $D^* = A \exp(\beta \Delta S_e)$ , where  $D^*$  are reduced transport coefficients, and  $A$  and  $\beta$  are scaling parameters that are very similar for systems with Coulombic potentials.<sup>6</sup> Consequently, the existence of an excess entropy anomaly is reflected in mobility anomalies. To make a more precise connection between thermodynamic and mobility anomalies, it is necessary to understand the mechanism underlying the scaling rule. Recently, scaling theories that diffusion in liquids takes place through a combination of binary collisions and cage dissociation. The binary collision contribution is approximately balanced out by using macroscopic evaluation parameters based on reptation theory. For example, reduced diffusivities are defined as  $D^* = D(\rho/\rho_{\text{eq}})^{1/2}$ . The fugacity of cage dissociation is assumed to be proportional to the number of accessible configurations, except at sites configurations space available is high in the stable liquid phase. For the scaling relationship to be state point independent, it is necessary that the dynamical parameter  $\beta$  controlling the number of accessible configurations is determined by the interaction potential and is otherwise state point independent.

This paper develops a basis for quantitative comparison of structure–entropy–diffusivity relationships in liquids with very different underlying interactions but similar water-like anomalies. We focus on three different categories of liquids: 1) molecular fluids (H<sub>2</sub>O), 2) tetrahedral ionic melt (BeF<sub>4</sub>) and

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