B.Sc. (Hons)/B.A. (Hons) (other than Mathematics) (Sem I)
GE-1(i): Fundamentals of Calculus

Total Marks: 100 (Theory: 75, Internal Assessment: 25) **Examination:** 3 Hrs.
Workload: 3 Lectures, 1 Tutorial (per week) **Credits:** 4 **Duration:** 14 Weeks

Course Objectives: Calculus is referred as 'Mathematics of change' and is concerned with describing the precise way in which changes in one variable relate to the changes in another. Through this course, students can understand the quantitative change in the behaviour of the variables and apply them on the problems related to the environment.

Course Learning Outcomes: The students who take this course will be able to:

i) Understand continuity and differentiability in terms of limits.
ii) Describe asymptotic behavior in terms of limits involving infinity.
iii) Understand the importance of mean value theorems and its applications.
iv) Learn about Maclaurin's series expansion of elementary functions.
v) Use derivatives to explore the behavior of a given function, locating and classifying its extrema, and graphing the polynomial and rational functions.

Unit 1: Continuity and Differentiability of Functions

Limits and continuity, Types of discontinuities; Differentiability of functions; Successive differentiation: Calculation of the *n*th derivatives, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

Unit 2: Mean Value Theorems and its Applications

Rolle's theorem, Mean value theorems and applications to monotonic functions and inequalities; Expansion of functions: Taylor's theorem, Taylor's series, Maclaurin's series expansion of $e^x, \sin x, \cos x, \log(1+x)$ and $(1+x)^m$; Indeterminate forms.

Unit 3: Tracing of Curves

Concavity and inflexion points, Asymptotes (parallel to axes and oblique), Relative extrema, Tracing graphs of polynomial functions, rational functions, and polar equations.

References:

1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). Wiley India Pvt. Ltd. New Delhi. International Student Version. Indian Reprint 2016.
2. Prasad, Gorakh (2016). *Differential Calculus* (19th ed.). Pothishala Pvt. Ltd. Allahabad.

Additional Reading:

i. Thomas Jr., George B., Weir, Maurice D., & Hass, Joel (2014). *Thomas*' *Calculus* (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

Teaching Plan (GE-1(i): Fundamentals of Calculus):

Weeks 1 and 2: Limits and continuity, Types of discontinuities.
[1] Chapter 1 (Theorems without proofs).
[2] Chapter 2 (Section 2.7).

Week 3: Differentiability of functions.
[1] Chapter 2 (Section 2.2).
[2] Chapter 3 (Section 3.2).

Week 4: Successive differentiation, Leibnitz theorem.
[2] Chapter 5.

Week 5: Partial differentiation, Euler's theorem on homogeneous functions.
[2] Chapter 12 [Section 12.2 (12.21 without proof, exclude 12.22 and 12.23), and Section 12.3].

Weeks 6 and 7: Rolle's theorem, Mean value theorems and applications to monotonic functions and inequalities.
[2] Chapter 7 (Sections 7.4 to 7.6).

Weeks 8 and 9: Taylor's theorem with Lagrange's and Cauchy's forms of remainders, Definition and examples of convergent sequences and series, Taylor's series, Maclaurin's series expansion of $e^x, \sin x, \cos x, \log(1+x)$, and $(1+x)^m$.
[2] Chapter 6 (Brief introduction of convergence from the Sections 6.1 and 6.2).
[2] Chapter 7 (Sections 7.7 and 7.8).

Week 10: Indeterminate forms.
[1] Chapter 6 (Section 6.5).
[2] Chapter 16 (Examples and Exercises).

Weeks 11 and 12: Concavity and inflexion points, Asymptotes (parallel to axes and oblique).
[1] Chapter 3 [Section 3.1 (3.1.3 to 3.1.5)].
[2] Chapter 9 (Sections 9.1 to 9.4).

Weeks 13 and 14: Relative extrema, Tracing graphs of polynomial and rational functions.
[1] Chapter 3 (Sections 3.2 and 3.3), and Chapter 10 (Section 10.2).

B.Sc. (Hons)/B.A. (Hons) (other than Mathematics) (Sem I)
GE-1(ii): Theory of Equations and Symmetries

Total Marks: 100 (Theory: 75, Internal Assessment: 25) **Examination:** 3 Hrs.
Workload: 3 Lectures, 1 Tutorial (per week) **Credits:** 4 **Duration:** 14 Weeks

Course Objectives: The goal of this paper is to acquaint students with certain ideas about integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial, and finding roots of cubic and quartic equations in special cases using elementary symmetric functions and in general using Cardon's and Descartes' methods, respectively.

Course Learning Outcomes: After completion of this paper, the students will be able to:

i) Understand the nature of the roots of polynomial equations and their symmetries.
ii) Solve cubic and quartic polynomial equations with special condition on roots and in general.
iii) Find symmetric functions in terms of the elementary symmetric polynomials.

Unit 1: Polynomial Equations and Properties

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the *n*th roots of unity and symmetries of the solutions.

Unit 2: Cubic and Biquadratic (Quartic) Equations

Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

Unit 3: Symmetric Functions

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric functions such as $\sum\alpha^2\beta,\ \sum\alpha^2\beta^2, \sum\alpha^2\beta\gamma, \sum\frac{1}{\alpha^2\beta\gamma}, \sum\alpha^{-3}, \sum(\beta+\gamma-\alpha)^2,\ \sum\frac{\alpha^2+\beta\gamma}{\beta+\gamma},\ldots$ of polynomial equations; Transformation of equations by symmetric functions and in general.

References:

1. Burnside, W.S., & Panton, A.W. (1979). *The Theory of Equations* (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
2. Dickson, Leonard Eugene (2009). *First Course in the Theory of Equations*. John Wiley & Sons, Inc. The Project Gutenberg eBook: http://www.gutenberg.org/ebooks/29785

Additional Reading:

i. Prasad, Chandrika (2017). *Text Book of Algebra and Theory of Equations*. Pothishala Pvt Ltd.

Teaching Plan (GE-1(ii): Theory of Equations and Symmetries):

Weeks 1 and 2: General properties of polynomials and equations; Statement of the Fundamental theorem of algebra and its consequences.
[1] Chapter I (Sections 8, 9 and 10); Chapter II (Sections 12 to 17).
[2] Chapter II (Sections 13 to 19)

Weeks 3 and 4: Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots.
[1] Chapter II (Sections 18 to 22).
[2] Chapter II (Sections 21, 24, 25 and 27), and Chapter VI [Section 67]
(Proofs of theorems in the Chapters II and VI are omitted).

Weeks 5 and 6: Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given.
[1] Chapter III (Sections 23 and 24).
[2] Chapter II (Sections 20).

Weeks 7 and 8: De Moivre's theorem for rational indices, the *n*th roots of unity and symmetries of the solutions; Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms.
[1] Chapter III (Section 26); Chapter IV (Sections 29 to 34).
[2] Chapter I (Sections 7 to 10).

Weeks 9 and 10: Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.
[1] Chapter VI (Sections 56 and 64).
[2] Chapter IV (Sections 42, 43, 51 and 52).

Weeks 11 and 12: Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots.
[2] Chapter IX (Sections 103 to 106, methods only).
[1] Chapter VIII (Section 77, method only).

Weeks 13 and 14: Computation of symmetric functions such as:
$\sum\alpha^2\beta,\ \sum\alpha^2\beta^2, \sum\alpha^2\beta\gamma, \sum\frac{1}{\alpha^2\beta\gamma}, \sum\alpha^{-3}, \sum(\beta+\gamma-\alpha)^2,\ \sum\frac{\alpha^2+\beta\gamma}{\beta+\gamma},\ldots$ of polynomial equations;
Transformation of equations by symmetric functions and in general.
[1] Chapter III (Sections 27 and 28); Chapter IV (Sections 39, 41 and 44).
[2] Chapter IX (Section 109, methods only).